

Laboratori Nazionali di Frascati

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R. Gatto: ELECTROMAGNETIC INTERACTIONS.

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I shall mainly talk of the following subjects:

- 1) Electron-nucleon scattering and nucleon-antinucleon annihilation into electron pairs: one-photon exchange; analysis and applications of two-photon terms;
- 2) Speculations about the photon as a Regge pole;
- 3) Form factors: dispersion theory calculations; consequences of symmetries, digression on unitary symmetry;
- 4) Electron scattering on deuterium,  $H^3$ , and  $He^3$ ;
- 5) Photoproduction;
- 6) Electroproduction.

### 1 - ELECTRON-NUCLEON SCATTERING AND NUCLEON-ANTINUCLEON ANNIHILATION INTO ELECTRON PAIRS -

In this section I shall summarize the theoretical framework for a discussion of electron scattering on nucleon and nucleon-antinucleon annihilation into an electron pairs.

We first consider one-photon exchange.

#### 1. 1. - One-photon exchange.

a) One-photon exchange corresponds to the graph of Fig. 1.

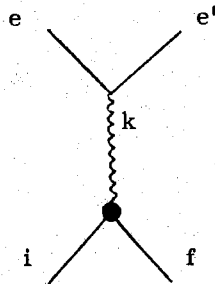


FIG. 1

The nucleon current is written as

$$\bar{u}_f (F_1(K^2) \gamma_\mu + \frac{1}{2M} F_2(K^2) \sigma_{\mu\nu} K_\nu) u_i \quad (1)$$

where  $u_f$  and  $u_i$  are Dirac spinors,  $\sigma_{\mu\nu} = 1/2 [\gamma_\mu, \gamma_\nu]$ ,  $K$  is the virtual photon momentum,  $M$  is the nucleon mass, and  $F_1$  and  $F_2$  are the so-called Dirac and Pauli form factors of the nucleon.

The normalization is:

$$\left. \begin{array}{l} F_1(0) = e \\ F_2(0) = eg_p \end{array} \right\} \text{ for the proton, and } \left. \begin{array}{l} F_1(0) = 0 \\ F_2(0) = eg_n \end{array} \right\} \text{ for the neutron}$$

with  $g_p = 1.79$  and  $g_n = -1.91$ . The isoscalar and isovector form factors are related to the proton and neutron form factors by

$$\left\{ \begin{array}{l} F_1^{(p)} = F_1^{(s)} + F_1^{(v)} \\ F_2^{(p)} = F_2^{(s)} + F_2^{(v)} \end{array} \right. \text{ and } \left\{ \begin{array}{l} F_1^{(n)} = F_1^{(s)} - F_1^{(v)} \\ F_2^{(n)} = F_2^{(s)} - F_2^{(v)} \end{array} \right.$$

The form factors  $F_1(K^2)$  and  $F_2(K^2)$  describe for space-like  $K^2 (K^2 > 0)$  electron-nucleon scattering, for  $K^2$  timelike and  $< -4M^2$  nucleon-antinucleon annihilation into an electron pairs.

The form factors are real for  $K^2 > -4\mu^2$  (where  $\mu =$  pion mass) and complex in general for  $K^2 < -4\mu^2$  (the so-called absorptive region). This is illustrated in the diagram of Fig. 2.

b) The Rosenbluth formula<sup>(1)</sup> gives the differential cross-section for electron-nucleon scattering.

In the laboratory system it can be written as

$$\frac{d\sigma}{d\Omega} = \sigma_0 \left\{ F_1^2(K^2) + \xi \left[ 2(F_1(K^2) + F_2(K^2))^2 \operatorname{tg}^2 \frac{\theta}{2} + F_2^2(K^2) \right] \right\} \quad (2)$$

where:

$$\sigma_0 = \frac{e^2}{4(4\pi)^2 E_1^2} \frac{\cos^2(\theta/2)}{\sin^4(\theta/2)} \frac{1}{1+(2E_1/M)\sin^2(\theta/2)}$$

$$\xi = \frac{K^2}{4M^2} \geq 0 \quad K^2 = \frac{(2E_1 \sin \theta/2)^2}{1+(2E_1/M)\sin^2 \theta/2} \geq 0 \quad (3)$$

In these expressions  $E_1$  is the electron laboratory energy and  $\theta$  is the laboratory scattering angle.

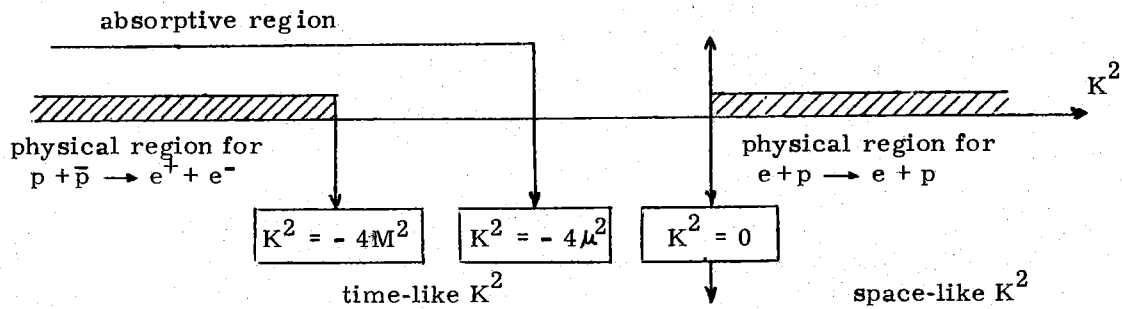


FIG. 2

c) Annihilation into an electron pair.

The following formula<sup>(2)</sup> gives in the center of mass system the differential cross-section for nucleon-antinucleon annihilation into an electron pair:

$$\frac{d\sigma}{d(\cos\theta_c)} = \frac{\pi_1^2}{8(4\pi)^2} \frac{1}{E\sqrt{E^2-M^2}} \left[ |F_1(K^2) + F_2(K^2)|^2 (1 + \cos^2\theta_c) + \left| \frac{M}{E} F_1(K^2) + \frac{E}{M} F_2(K^2) \right|^2 \sin^2\theta_c \right] \quad (4)$$

where  $\theta_c$  is the c. m. angle,  $E$  is the total proton energy in c. m. and

$$K^2 = -4E^2 < 4M^2 \quad (5)$$

The colliding beam reaction  $e^+ + e^- \rightarrow p + \bar{p}$ <sup>(3)</sup> is just the inverse of  $p + \bar{p} \rightarrow e^+ + e^-$  and its cross-section is given by (4) multiplied by  $\beta^3$ , where  $\beta$  is the velocity of the proton in c. m.

d) Charge and magnetization form factors.

In Eq. (4) the form factors appear in two particular linear combinations. Defining:

$$G_E(K^2) = F_1(K^2) - \xi F_2(K^2) \quad (6)$$

$$G_M(K^2) = F_1(K^2) + F_2(K^2) \quad (6')$$

with  $\xi$  given by Eq. (3), the annihilation formula becomes

$$\frac{d\sigma}{d(\cos\theta_c)} = \frac{\pi}{8} \frac{e^2}{(4\pi)^2} \frac{1}{E\sqrt{E^2-M^2}} \left[ |G_M|^2 (1 + \cos^2\theta_c) - \frac{1}{\xi} |G_E|^2 \sin^2\theta_c \right] \quad (4')$$

The same linear combinations (6) and (6') bring the Rosenbluth formula in the form:

$$\frac{d\sigma}{d\Omega} = \sigma_0 \left\{ \frac{G_E^2 + \frac{2}{3} G_M^2}{1 + \frac{2}{3}} + 2 \frac{2}{3} G_M^2 \tan^2 \frac{\theta}{2} \right\} \quad (2')$$

It is remarkable that only the squares of  $G_E$  and  $G_M$  appear in (2').

Sachs<sup>(4)</sup> shows that  $G_E$  and  $G_M$  can be thought of as describing the electric and magnetic distributions of the nucleon.

More precisely: the Fourier transforms of  $G_E$  and  $G_M$  (taken in the Breit system, where  $K_0 = 0$ ) can be considered as the spatial distributions of charge and of magnetization (they describe the interactions with a static electric field and a static magnetic field respectively).

e) The form factors in the time-like region.

Numerical estimates of the cross-section for  $p + \bar{p} \rightarrow e^+ + e^-$  depend very sensitively on the unknown values of the form factors at negative  $K^2$ . Extrapolations from the space-like region could well turn out to be quite unreliable. Note that the value of the total annihilation cross section at energy  $E$ , for the incident antiproton in c. m., depends on the values of the form factors at the point  $K^2 = -4E^2 > -4M^2$ .

An exact calculation of the nucleon form factors in the lower energy part of the physical region for  $p + \bar{p} \rightarrow e^+ + e^-$  can be performed under the following assumptions. In the dispersion relations for the form factors one adds to the contribution from intermediate multipion states the contribution from intermediate nucleon-antinucleon states, which is expected to be important in this region (Fig. 3).

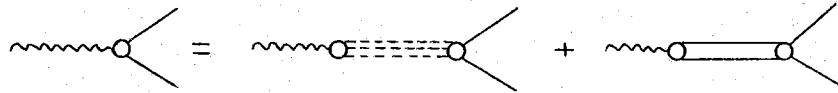


FIG. 3

One assumes that the multipion contribution can be approximated by extrapolation of the empirical form factors of the space-like region. More precisely, one assumes that the same pionic resonances which dominate the behaviour of the form factors in the space-like region are still the dominant contributors among the mesonic intermediate states in the time-like region of physical interest. One is then led to a system of inhomogeneous integral equations for the form factors (see Fig. 3). The solution depends however on the proton-antiproton scattering phase-shifts for  ${}^3S_1 \leftrightarrow {}^3S_1$ ,  ${}^3D_1 \leftrightarrow {}^3D_1$  and  ${}^3S_1 \leftrightarrow {}^3D_1$ , which appear in the expression of the kernel of the integral equation. One hopes that some information on such phase shifts could soon be available to make possible some definite predictions on this important problem.

Angular distribution experiments will separately determine  $|G_M|^2$  and  $|G_E|^2$  for the proton. The relative phase between  $G_M$  and  $G_E$  can be determined by performing experiments with polarized antiproton beams, or experiments of annihilation on polarized proton targets.

Consider an experiment with polarized antiprotons.

The antiproton polarization vector is called  $\vec{P}$ ; the unit vector normal to the plane of the reaction is called  $\vec{n}$ . The angular distribution contains now a term depending on the cosine of the angle between  $\vec{P}$  and  $\vec{n}$ :

$$\frac{d\sigma}{d(\cos\theta_c)} = \frac{d\sigma}{d(\cos\theta_c)} \Big|_{\text{unpolarized}} + \frac{M}{E} \text{Im}(G_E^* G_M) |\sin 2\theta_c| (\vec{P} \cdot \vec{n}) \quad (22)$$

The new term is proportional to the sine of the phase difference between  $G_E$  and  $G_M$ . In fact if the form factors were in phase the new term should vanish as directly implied by time reversal invariance ( $\vec{P}$  changes sign under time reversal, while  $\vec{n}$  stays invariant).

If the experiment is carried out with unpolarized antiprotons on a polarized proton tar

get, with polarization  $\vec{P}$ , the same Eq. (22) applies except for a minus sign in front of the last term.

Dispersion relations can be used to obtain information on the phases of  $G_E$  and  $G_M$ .

Let us write, for instance,  $G_E = |G_E| e^{i\delta_E}$ . The dispersion relation:

$$\text{Re}G_E(K^2) = \frac{1}{\pi} P \int_{-\infty}^{-4\mu^2} \frac{dm^2 \text{Im}G_E(m^2)}{m^2 - K^2} \quad (23)$$

can be written in the form

$$|G_E(K^2)| \cos \delta_E(K^2) = \frac{1}{\pi} P \int_{-\infty}^{-4\mu^2} \frac{dm^2 |G_E(m^2)| \sin \delta_E(m^2)}{m^2 - K^2} \quad (24)$$

Eq. (24) can be thought of as an integral equation for the phase  $\delta_E(K^2)$  assuming  $|G_E(K^2)|$  to be known.

Freund and Kummer<sup>(5)</sup> have shown that, provided  $G_E(K^2)$  is analytic in the cut plane, has no complex zeros, and satisfies  $G_E(z^*) = G_E^*(z)$  one obtains

$$\delta_E(K^2) = -\frac{K^2}{2\pi} P \int_{-\infty}^{+\infty} \frac{\log |G_E(m^2)| dm^2}{(K^2 - m^2)m^2} \quad (25)$$

The same of course holds for  $G_M(K^2)$ .

The integral in Eq. (25) can be split into three integrals: one from  $-\infty$  to  $-4M^2$  (physical region for  $p + \bar{p} \rightarrow e^+ + e^-$  or  $e^+ + e^- \rightarrow p + \bar{p}$ ), one from  $-4M^2$  to 0 (unphysical region), and one from 0 to  $+\infty$  (physical region for  $e + p \rightarrow e + p$ ). For the values of  $|G_E(K^2)|$  and  $|G_M(K^2)|$  in the unphysical region one has to make guesses (assuming for instance that resonances dominate that region).

However, out of such guesses, and of the measured values of  $|G_E(K^2)|$  and  $|G_M(K^2)|$ , one must obtain  $\delta_E(K^2)$  and  $\delta_M(K^2)$  such that: they are zero everywhere, except in the absorptive region (i. e. for  $K^2$  from  $-\infty$  to  $-4\mu^2$ ); the phase difference  $\delta_E(K^2) - \delta_M(K^2)$  must fit the polarization measurements.

f) Annihilation into a muon pair as a test of electrodynamics.

Eqs. (4) and (4') have been derived assuming  $m_e = 0$ . For finite mass one has simply to make the following substitutions in (4'): multiply the cross-section by an overall factor  $\beta$  (velocity of the final lepton), and substitute according to  $(1 + \cos^2\theta_c) \rightarrow (2 - \beta^2 \sin^2\theta_c)$ ,  $\sin^2\theta_c \rightarrow (1 - \beta^2 \cos^2\theta_c)$ .

The ratio of the total cross-section for  $p + \bar{p} \rightarrow \mu + \bar{\mu}$  to the total cross-section for  $p + \bar{p} \rightarrow e + \bar{e}$ , then comes out to be

$$\frac{\sigma_t(p\bar{p} \rightarrow \mu\bar{\mu})}{\sigma_t(p\bar{p} \rightarrow e\bar{e})} \simeq 1 - \frac{3}{8} \left(\frac{m_\mu}{E}\right)^4 \quad (26)$$

which is almost exactly 1 over the whole energy range.

The quantum-electrodynamical radiative corrections to (26) have been calculated, but they seem to lie beyond the expected experimental accuracy.

The measurement of the  $(2\mu)/(2e)$  ratio in  $p\bar{p}$  annihilation would provide a very accurate test of quantum electrodynamics and of muon structure. The momentum transfers are larger than  $2M$  and time-like. Tests of this kind have never been carried out so far.

g) Annihilation into intermediate bosons.

If intermediate bosons exist the annihilation mode

$$p + \bar{p} \rightarrow B + \bar{B}$$

where B is an intermediate vector boson will occur at sufficiently high energy.

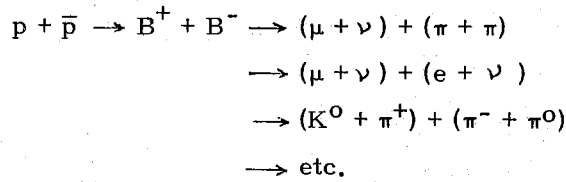
The ratio:

$$b = \frac{\sigma_T(p\bar{p} \rightarrow B\bar{B})}{\sigma_T(p\bar{p} \rightarrow e^+e^-)}$$

depends on a bilinear combination of the electromagnetic form factors of B.

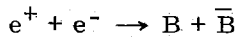
If B has no anomalous moments and constant form factors, b is simply  $b = \sqrt{3} \frac{3}{4} + (E/m_B)^2$ . In such a case, annihilation into a pair of intermediate bosons is favored with respect to annihilation into  $e^+e^-$  or  $\mu^+\mu^-$  already for c. m. energy larger than  $1.5 m_B$ .

The meson B is expected to decay fastly into  $2 \pi$ ,  $3 \pi$ ,  $4 \pi$ ,  $\pi + K$ ,  $\mu + \nu$ ,  $e + \nu$ , etc. The annihilation events into  $B + \bar{B}$  will in some cases be of the kind



and, in general, they will exhibit definite correlations.

If intermediate bosons exist the reaction



is expected to be a very convenient mode of producing them. We shall not review here the possibilities of electron-positron colliding beams, now under construction, and the related theoretical work.

1. 2 - Higher electromagnetic orders. Two photon terms.

a) The radiative corrections to electron-proton scattering were estimated by Tsai<sup>(6)</sup>, with some important qualifications that we shall here discuss in more detail.

The graphs of Fig. 4 must be included in the calculation of the elastic cross-section (the thin line denotes an electron, the heavy line a nucleon):

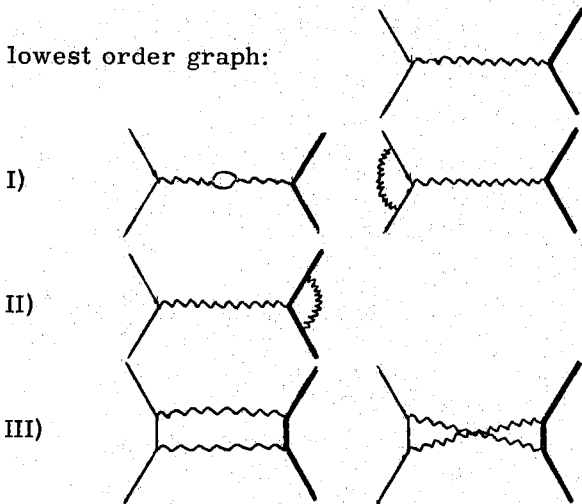


FIG. 4

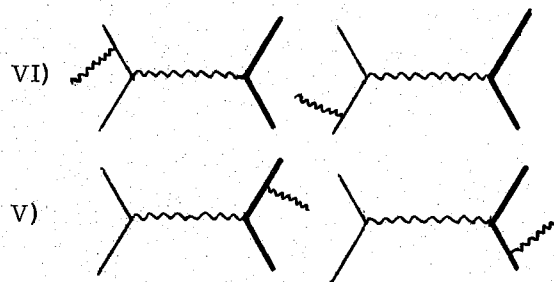


FIG. 5

Furthermore one must add incoherently inelastic (bremsstrahlung) contributions from the graphs of Fig. 5.

All such graphs must be corrected for virtual meson effects.

Virtual mesonic effects make the evaluation of the graphs (III) very difficult: We shall

refer to these graphs as to the "two-photon terms".

In Tsai's calculation only the infrared parts of these terms are evaluated. In the absence of mesonic effects the extraction of the infrared parts would be justified. Non-infrared parts would in fact be negligible.

Virtual meson effects are present also in (II). However they only produce an (expectedly small) modification of the momentum dependence in the form factors (inclusion of (II) does not alter the Rosenbluth formula).

Similarly in the proton bremsstrahlung graphs (V) mesonic effects are not expected to play a relevant role. Tsai's calculations are reported to be valid up to 5 BeV within 1% if one assumes that no errors arise from the treatment of the mesonic effects.

This last assumption is highly questionable.

On the other hand no reliable calculations exist at present of the mesonic effects. One needs a reliable evaluation of the two-photon terms, corresponding to the graphs (III) with all mesonic contributions included.

The problem was considered by Drell and Ruderman<sup>(7)</sup> and Drell and Fubini<sup>(8)</sup> who tried to evaluate the effect of the 3-3 resonance.

b) The 3-3 resonance contribution to the two-photon term.

By direct application of the reduction formula, to order  $e^4$ .

Drell and Fubini write the amplitude for electron-proton scattering in the form

$$(2\pi)^4 \delta(p_1+q_1 - p_2-q_2) \bar{u}(q_2) \gamma_\mu u(q_1) \langle p_2 | A_\mu | p_1 \rangle - \frac{1}{2} e^2 \int d^4x d^4y e^{iq_2x} \bar{u}(q_2) \gamma_\mu S_F(y-x) \cdot \gamma_\nu u(q_1) e^{-iq_1y} \langle p_2 | P [\bar{A}_\mu(y) A_\nu(x)] | p_1 \rangle. \quad (27)$$

where:  $p_1$ ,  $p_2$ ,  $q_1$  and  $q_2$  are initial and final proton and electron momenta respectively, the spinors  $u$  describe the electron, and  $A_\mu$  is the electromagnetic field. The amplitude is thus split into a one-photon part (the first term in (27)) and a two-photon part (the second term in (27)) (Fig. 6).

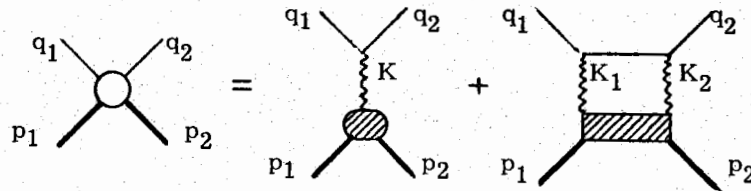


FIG. 6

In addition to the lowest order amplitude the one-photon term contains contributions corresponding to the graph (II) which are still one-photon-exchange terms.

One sees that the two-photon terms are describable in terms of the Compton scattering of a virtual photon a physical nucleon.

The two-photon term in (27) can be written in momentum space as:

$$ie^2(2\pi)^4 \delta(p_1+q_1-q_2) \sum_{\text{pol.}} \int M_{\mu\nu} E_{\mu\nu} d^4K_1 \quad (28)$$

where  $M_{\mu\nu}$  is proportional to the Compton matrix element for virtual photons,  $E_{\mu\nu}$  is formed out of known quantities, and the integration is on the virtual photon momentum  $K_1$ .

Drell and Fubini assume that, for low energies,  $M$  can be expressed in terms of the amplitude for real Compton scattering, simply by multiplying the real amplitude by  $F_m(K_1^2) F_m(K_2^2)$  where  $F_m$  is the magnetic form factor of the nucleon and  $K_1$  and  $K_2$  are the virtual photon momenta. This procedure was shown to be approximately valid in the analysis of pion

electroproduction by Fubini, Nambu and Wataghin.

The main point is then the following.

The low-energy Compton scattering amplitude for a photon of center-of-mass energy  $\omega$  is dominated by a resonant term proportional to

$$\frac{1}{\omega - \omega_R + i\Gamma} \quad (29)$$

where  $\omega_R$  is the energy of the 3-3 resonance and  $\Gamma$  its width.

The real part of (29) changes sign in passing through the resonance. The imaginary part of (29) is instead positive in the whole region.

Because of the change in sign of the real part when passing through the resonance we expect that the contribution of the real part to the integral in (28) be small. This is indeed verified by the detailed calculation of Drell and Fubini.

In the scattering amplitude the one-photon term is real. To lowest order the two-photon term contributes to the differential cross-section through its interference with the real one-photon term. Thus we need to know only the real part of the two-photon term. However, as we have explained, the contribution from the 3-3 resonance to the real part of the two-photon term is expected to be very small.

Drell and Fubini calculate a 1% increase of the cross-section in the energy range  $\approx 1$  BeV from the interference between the two-photon term and the one-photon term.

Although the calculation is strictly non-relativistic, the general feature we have found, that resonances due to nucleon isobars produce contributions to the real part of the two-photon amplitude that are odd in  $\omega$ , is also valid for other pion-nucleon resonances. On this basis we do not expect large effects from the nucleon resonances in the differential cross-section.

On the same basis however there is no reason to expect that also the contributions to the imaginary part of the two-photon term be small. The imaginary part is essentially proportional to the polarization of the recoil nucleon in electron-nucleon scattering. We shall come back later on this question.

### c) Calculation of nucleon polarizability.

A different calculation of the two-photon terms based on a Weizsacker-Williams approximation has been performed by Werthamer and Ruderman<sup>(9)</sup>. The electron-nucleon scattering amplitude is expanded in partial waves (spin effects are neglected)

$$F(\theta) = (2ik)^{-1} \sum_l (2l+1) \left[ e^{2i\delta_l} - 1 \right] P_l(\cos\theta) \quad (30)$$

In (30)  $\theta$  is the scattering angle,  $K$  is the electron momentum, and the phaseshift  $\delta_l$  are in general complex. Assuming each  $\delta_l$  to be small one can write for the imaginary part of  $F(\theta)$

$$I_m F(\theta) = \frac{K}{4\pi} \sum_l \sigma_l^{(R)} P_l(\cos\theta) \approx \frac{K}{4\pi} \int_0^\infty \sigma^{(R)}(\rho) P_{K\rho}(\cos\theta) d\rho \quad (31)$$

where  $\sigma_l^{(R)}$  is the reaction cross section for the  $l$ -wave, and one has, in the spirit of a WKB approximation, substituted an integral for the sum over  $l$  and introduced the impact parameter  $\rho = l/K$ .

The partial cross-section  $\sigma^{(R)}(\rho)$  is mainly due to meson production by the electromagnetic field of the incident electron. Werthamer and Ruderman use a Weizsacker-Williams approximation to evaluate  $\sigma^{(R)}(\rho)$  as:

$$\sigma^{(R)}(\rho) = 2\pi \int_0^{\bar{K}} \rho N(\rho, \omega) \sigma_{\gamma\pi}(\omega) d\omega \quad (32)$$

where  $K$  is the incident electron momentum in the laboratory system,  $N(\rho, \omega)$  is the effective flux of real photons from an electron with impact parameter  $\rho$  and  $\sigma_{\gamma\pi}(\omega)$  is the total



photoproduction cross-section from real photons of frequency  $\omega$  in the laboratory system. For  $N(\rho, \omega)$  the authors use an improvement of the Weizsacker-Williams expression.

The theory gives, in the forward direction,  $\text{Im } F(0)$  increasing with energy like  $\text{KlnK}$ .

This forward imaginary part generates a real part by a dispersion relation. The real part also comes out to be monotonic and of magnitude comparable to the imaginary part.

One finds that already at 1 BeV the forward contribution from such two-photon effects is already of the same order of the observed amplitude for large momentum transfer. In fact at higher energies one would expect such  $e^4$  terms to be large than the  $e^2$  amplitude at  $90^\circ$ . However from (31) and (32), by inserting the appropriate expression for  $N(\rho, \omega)$ , and by deriving the real part from a dispersion relation, one finds that this contribution is very rapidly decreasing with increasing incident momentum for any non-zero scattering angle.

The effective dependence cannot be stated explicitly as it depends on a form factor for meson photoproduction that is introduced in the calculation to correct the photomeson amplitude for virtual photons. However it can reasonably be stated that the two-meson effects calculated through this procedure contribute negligibly in comparison to the Born term.

d) Polarization of the recoil proton and asymmetry from polarized leptons.

For unpolarized incident electrons on unpolarized protons, the recoil proton polarization, in electron-proton scattering, is zero in the one-photon approximation. The polarization of the recoil proton is proportional to the product of the imaginary part of the two-photon term and the one-photon term.

Guerin and Piketty<sup>(12)</sup> have performed a calculation of the imaginary part of the two-photon amplitude by taking into account the contributions from the first and second pion-nucleon resonances,  $N^*$  at 1.238 MeV and  $N^{*K}$  at 1.512 MeV.

The imaginary part of the two-photon amplitude can directly be expressed by the unitarity relation in terms of the amplitudes for inelastic electron scattering (electroproduction of pions). Such amplitudes are calculated with the aid of the isobars  $N^*$  and  $N^{*K}$ .

The transition leading to  $N^*$  is essentially an isovector magnetic dipole transition, whereas the transition leading to  $N^{*K}$  is essentially an electric dipole transition which can contain a mixture of both isoscalar and isovector.

The hypothesis is made that the electromagnetic  $N$ - $N^*$  form factor is proportional to the vector magnetic nucleon form factor, and a similar, less justified, hypothesis is made for the  $N$ - $N^{*K}$  electromagnetic form factor.

The results obtained by the authors show a very small polarization for the recoil proton ( $\sim 0.3\%$  or less) for incident electron laboratory momenta up to 1 BeV/c.

Closely related to the recoil proton polarization is the fractional asymmetry in the differential cross-section from polarized electrons (or muons).

If the incident electrons (or muons) are polarized the scattered distribution will exhibit an azimuthal dependence also arising from the interference between two-photon exchange and the one-photon exchange.

The asymmetry parameter  $Y$  is defined from the expression  $1 + PY \cos\psi$  for the distribution in the azimuthal angle  $\psi$ , defined as the angle between the scattering plane and the plane normal to the electron (or muon) spin,  $P$  being the initial polarization.

The values calculated for  $Y$  are also very small, reaching a maximum of 0.5%, at a particular angle for incident laboratory momenta less than 1 BeV/c.

e) Contribution of meson resonances to the two-photon term.

Much attention has been given to the problem of the contributions from possible meson resonances to the two-photon terms, as illustrated in the graph of fig. 7.

The kind of deviation from the Rosenbluth formula that would originate from the interference between such a contribution and the lowest order term can be discussed phenomenologically, following arguments due to Gourdin and Martin<sup>(10)</sup>.

In fact if one looks at the process in the annihilation channel,  $p+p \rightarrow e^+e^-$ , one sees that the interaction through the resonant states occurs only for definite angular momentum and parity.

The lowest order contribution, due to single photon exchange, also takes place through definite angular momentum and parity, namely  $1^-$ .

Such features allow a direct discussion of the general form of the angular distribution to be expected for given spin and parity of the resonant mesonic state.

As a first case we shall examine the problem of the interference between the lowest order term represented by the graph of fig. 8 and a contribution of the kind (fig. 9) where a particle with definite spin  $J$  and parity  $P$  is directly exchanged between the proton and the electron.

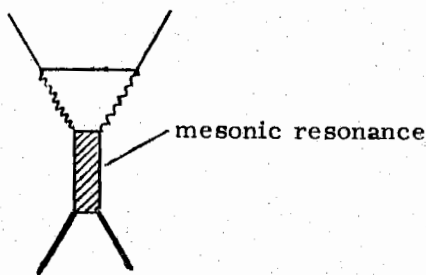


FIG. 7

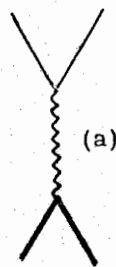


FIG. 8

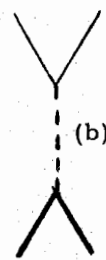


FIG. 9

The coupling of such a particle to the electron can be through photons or it could be a hypothetical direct coupling. Although there is no evidence so far of any other coupling between the proton and the electron except that through one or more photons, the question of knowing what kind of deviations from the Rosenbluth formula would occur if such a different coupling, through a particle of given spin and parity, exists, has often been asked.

Let us first discuss the obvious case of  $J=1$ ,  $P=-1$ , in which we consider the interference between the one-photon graph (a) and the graph (b) where the exchanged particle has  $J=1$ ,  $P=-1$ . In such a case in the annihilation channel the interaction goes, for both graphs, through states with  $J=1$  and  $P=-1$ , and one again obtains the Rosenbluth formula.

If we call  $\psi$  the center of mass angle in the annihilation channel and  $\theta$  the scattering angle (in the scattering channel) in the laboratory system, it can be easily verified that

$$\cos \psi = \left[ 1 + \frac{1}{1+\xi} \cotg^2 \frac{\theta}{2} \right]^{\frac{1}{2}} \quad (33)$$

where  $\xi$  had been defined in eq. (3), and  $\xi = K^2/4M^2$ .

The c. m. angular distribution in the annihilation channel, since the reaction goes through the single angular momentum and parity  $J=1$ ,  $P=-1$ , is as well known, of the form

$$a(K^2) + b(K^2) \cos^2 \psi \quad (34)$$

The angular distribution in the scattering channel can, according to the general arguments by Gourdin and Martin, be read off from the angular distribution (34) and the substitution (33). One finds by direct substitution

$$\cotg^2 \frac{\theta}{2} \left[ A(K^2) + B(K^2) \tg^2 \frac{\theta}{2} \right] \quad (35)$$

The term in the bracket is the same one occurring in the Rosenbluth expression.

In this way one also learns that the Rosenbluth linear dependence from  $\tg^2(\theta/2)$  is in fact more general than it would appear from the usual derivation: it holds for any spin of the target (for instance it holds in elastic processes such as  $e+d \rightarrow e+d$ , and in general for  $e + (\text{nucleus}) \rightarrow e + (\text{nucleus})$ , with the remark that for a spin zero target  $B=0$  (as follows from additional considerations).

A Rosenbluth linear  $\text{tg}^2(\theta/2)$  dependence also holds for inelastic processes (such as  $e + d \rightarrow n + p + e$ ) provided only the final electron is observed.

f) Axial vector resonance.

Let us now consider the case  $J = 1, P = +1$ . For such a case the angular distribution in the annihilation channel takes the general form:

$$a(K^2) + c(K^2) \cos \varphi + b(K^2) \cos^2 \varphi \quad (36)$$

The distribution (36) differs from the distribution (34) because of the linear term in  $\cos \varphi$ , that is present here because of the opposite parities of the intermediate states in graphs (a) and (b). Instead of (35) we now get the dependence

$$\cotg^2 \frac{\theta}{2} \left\{ A(K^2) + B(K^2) \text{tg}^2 \frac{\theta}{2} + \text{tg}^2 \frac{\theta}{2} C(K^2) \left[ \frac{1}{1+} \cotg^2 \frac{\theta}{2} \right]^{1/2} \right\} \quad (37)$$

In (37) one can approximate the whole expression in the bracket with a linear function of  $\text{tg}^2(\theta/2)$  in the whole range  $\text{tg}^2(\theta/2) > 2$ .

Thus a deviation from a linear Rosenbluth plot may only be seen at small angles. On the other hand, on physical grounds, we would expect deviations from the Rosenbluth formula only at large momentum transfers. To make the small angle deviations apparent one has to make experiments at very high energy.

In graph (b) the coupling of the hypothetical  $J = 1, P = +1$  particle to the electron line can occur through two-photon exchange provided the charge conjugation quantum number of the particle is  $C = +1$ . It must be noted that the coupling will vanish when the two photons become real because of the known selection rule that forbids a spin-one particle to go into two photons. Of the possible decay modes of such a particle the  $2\gamma$ ,  $\pi^0 + \gamma$ , and  $2\pi$  decay modes would be forbidden.

g) The other case to be discussed is that in which one adds to the lowest order one-photon graph (a) possible graphs of the kind (b) with exchanged  $0^+$  and  $0^-$  particles.

For such a case one can show that the Rosenbluth linear dependence is again fully reproduced because of a suppression of the interference between  $0^+$  and  $1^-$  that comes about from helicity considerations.

Furthermore if the coupling of the hypothetical particles to the electron is mediated by electromagnetic interactions one can show that in the limit of zero electron mass the coupling will vanish because of the property of the electromagnetic interaction to preserve helicity.

Similar considerations show that there can be no contribution from a  $2^-$  intermediate particle.

We will discuss the case  $2^+$  in more detail following a treatment by Flamm and Kummer<sup>(11)</sup>.

The general form of the scattering amplitude for electron-proton scattering can be written in terms of six independent invariants: for instance, the five Fermi invariants and one derivative coupling. In the limit of zero electron mass, among the Fermi couplings only the vector and the axial contribute, because of the property of electrodynamics to preserve helicity.

The general amplitude for electron-proton scattering can thus be written as:

$$T = a_1(\bar{e}\gamma_\mu e)(\bar{p}\gamma_\mu p) + a_2(\bar{e}\gamma_\mu\gamma_5 e)(\bar{p}\gamma_\mu\gamma_5 p) + a_3(\bar{e}\tilde{Q}e)(p\tilde{Q}p) \quad (38)$$

where  $a_1, a_2, a_3$  are invariant functions of the energy and the momentum transfer and  $\tilde{Q}$  can be chosen as  $\tilde{Q} = Q/|Q|$ , where  $a = P - (PR)R/R^2$ , with  $P = 1/2(P_1 + P_2)$  and  $R = 1/2(q_1 + q_2)$ , and with  $p_1, p_2, q_1$  and  $q_2$  initial and final proton and electron momenta respectively.

The general description (38) of electron-proton scattering is to be contrasted with the much simpler description in terms of the charge and magnetic form factors of the proton which

is valid however only in the one-photon approximation.

The two form factors are functions of the momentum transfer alone, whereas the three invariants  $\mathcal{A}_i$  in (38) are functions of both energy and momentum transfer. For the functions  $\mathcal{A}_i$  one can write a dispersion relation in the momentum transfer and keep in the absorptive part only one-photon and two-photon states.

The absorptive two-photon parts are expressed in terms of the Compton scattering amplitudes. There are six independent invariants describing real Compton scattering on a nucleon, but only three combinations of the six Compton invariant functions appear in the absorptive parts of the  $\mathcal{A}_i$ .

The assumption is made that the Compton amplitudes are those produced by a mesonic resonant state of given spin and parity.

Of course, the intermediate two-photon state inserted in the unitarity relation is a physical state, therefore the resonance spin cannot be one, according to the selection rule forbidding decay of a spin one particle into two photons.

Spin zero and spin two lead to a non-vanishing Compton amplitude; however those particular combinations of the Compton invariant functions that occur in the absorptive parts of the  $\mathcal{A}_i$  turn out to be zero for both cases of zero spin and for spin-parity  $2^-$ .

That this must happen is also directly clear by helicity arguments applied in the limit of vanishing electron mass.

h) Tensor resonance.

In conclusion only  $2^+$  can contribute as long as  $J \leq 2$  for the resonant particle.

Flamm and Kummer calculate the differential cross-section by simulating the two-photon term going through a  $2^+$  resonance with a direct coupling of a tensor particle to the electron and to the proton. The following local couplings are assumed for the coupling to the nucleon and to the electron respectively.

$$H_N = g_1 \frac{1}{2} \left\{ (\partial_\mu \bar{\psi})(\partial_\nu \psi) + (\partial_\nu \bar{\psi})(\partial_\mu \psi) \right\} B_{\mu\nu} + \\ + g_2 \frac{i}{8} \left\{ [\bar{\partial}_\nu \bar{\psi}, \gamma_\mu \psi] - [\bar{\psi} \gamma_\mu, \partial_\nu \psi] + (\mu \leftrightarrow \nu) \right\} B_{\mu\nu} \quad (39)$$

$$H_e = e^4 g_e \frac{i}{8} \left\{ [\bar{\partial}_\nu \bar{\psi}, \gamma_\mu \psi] - [\bar{\psi} \gamma_\mu, \partial_\nu \psi] + (\mu \leftrightarrow \nu) \right\} B_{\mu\nu} \quad (40)$$

where  $\psi$ ,  $\bar{\psi}$  and  $B$  describe the nucleon, the electron and the tensor particle respectively. The differential cross-section in the laboratory system takes the form

$$d\sigma = d\sigma_{\text{Rosenbluth}} + d\sigma_{\text{Mott}} \frac{e^2 g_e t}{1 - \frac{t_r}{t}} \cdot \frac{1}{4} \left[ 1 - \frac{4M^2}{t} - \right. \\ \left. - \frac{4M^2}{t} \text{ctg}^2 \frac{\theta}{2} \right]^{1/2} \left\{ 4MG_E g_1 - g_2 \left[ \bar{G}_M + F_1 + \frac{t}{M^2} G_M \text{tg}^2 \frac{\theta}{2} \right] \right\} \quad (41)$$

where  $t$  is the momentum transfer,  $t = -K^2$ , and  $t_r$  correspond to the resonance mass.

It can be seen from (41) that a deviation from the linear  $\text{tg}^2(\theta/2)$  dependence occurs mainly at small angles. Flamm and Kummer remark that to have a 10% deviation at  $t = -30 \text{ f}^{-2}$  from the Rosenbluth formula the coupling constants must be of order unity if  $t_r$  is of the order of the known resonance masses.

However such a 10% deviation from the Rosenbluth cross-section would correspond to a much smaller deviation from the linear Rosenbluth dependence at finite angles ( $\sim 3.5\%$  for  $\text{tg}^2(\theta/2) \approx 1$ ).

Again one would need experiments at small angles, implying, for sufficiently large momentum transfers, very high energy.

## i) Summary.

We summarize the conclusions about the two-photon exchange terms and their possible experimental and theoretical analysis in both electron-proton scattering and proton-antiproton annihilation into electrons.

The real part of the two-photon exchange amplitude interferes with the real one-photon amplitude in the differential cross-section for electron-proton scattering, giving rise to deviations from the Rosenbluth formula.

The contributions to such a real part from resonances in the scattering channel (nucleon isobars) are expected to be small, partly because of a oddness of the real part of the resonant amplitude about the resonant energy.

Contributions to the two-photon amplitude (both real and imaginary part) from mesonic resonances in the annihilation channel can only originate from a  $1^+$  resonance or a  $2^+$  resonance, (excluding larger spins).

The spin-parity assignment  $1^+$  to the so-called Buddha particle ( $\omega$ - $\pi$  resonance) is not inconsistent at this moment with the scanty experimental data.

Also a  $2^+$  assignment is possible for the  $f^0$  meson.

In both cases of an interference of the one-photon term with a two-photon term dominated by a  $1^+$  or  $2^+$  resonance, deviations from the Rosenbluth linear  $\text{tg}^2(\theta/2)$  behaviour are only expected at small angles, implying the necessity of measurements at high energies.

The deviations from the Rosenbluth formula arising from the interference between one-photon exchange and two-photon exchange are apposite in sign for electron and positron. In fact the two-photon terms have the same sign for both electron and positron, whereas the one-photon term changes sign.

A direct way of observing the presence of two-photon exchange is thus the observation of a difference between electron and positron cross-section (after subtraction of the standard radiative corrections).

The imaginary part of the two-photon exchange term gives rise to a polarization of the recoil proton in electron-proton scattering from unpolarized initial particles. Direct calculations of the imaginary part by direct use of unitarity and with different models show however that this polarization should be small.

In proton-antiproton annihilation into electrons already the one-photon exchange term is expected to be complex, because the form factors are in the absorptive region. Therefore the separation of the two-photon terms into real and imaginary part is less relevant to the physical discussion.

In particular the differential cross-section depends on the proton (or antiproton) polarization already at the lowest electromagnetic order.

However the presence of two-photon exchange in proton-antiproton annihilation can also be readily detected by observing possible asymmetries with respect to the final  $e^+$  and  $e^-$ .

As long as one performs experiments that are symmetric with respect to the final electron and positron (for instance experiments that do not distinguish the electron from the positron) there can be no interference from two-photon exchange.

However two-photon exchange will generally give rise to asymmetries between the final electron and the final positron that, if detected by the apparatus, would suggest the presence of two-photon terms.

## 2 - SPECULATIONS ABOUT THE PHOTON AS A REGGE POLE -

## a) The photon as a Regge pole.

The experimental analyses carried out after the proposal by Blankenbecler, Cook and

Goldberger<sup>(13)</sup>, that the photon lies on a Regge trajectory, have, at this time, greatly reduced the original interest in the subject. The subject is however related to relevant theoretical work and it may be worthwhile to examine it more closely.

Blankenbecler, Cook and Goldberger based their proposal on the argument that, since the photons interact with strong interacting particles, they must be non-elementary if strong interacting particles are non-elementary. Furthermore, if strong interacting particles are treated as Regge poles, whereas the photon is treated as elementary, then photon interactions will eventually dominate over strong interactions in a scattering process, when energy is increased and momentum transfer is kept constant. A simple estimate for proton-proton scattering shows that the Regge matrix element due to the vacuum pole is dominated by the lowest order electromagnetic matrix element at a laboratory energy of  $\sim 340$  BeV for a momentum transfer of  $\sim 50 \mu^2$ . The two matrix elements become comparable already at lower energies and one would expect quite large electromagnetic corrections even at the energies that can presently be reached with machines.

Blankenbecler, Cook and Goldberger make the assumption that a Regge trajectory is associated with the photon. The photon trajectory  $\alpha_1(t)$ , must have value 1 at  $t = 0$  and must have odd signature. Its slope at the origin is unknown.

If it turns out to be comparable to the slope of the vacuum trajectory one would speak of a "strong reggeization" of the photon, if it turns out to be much smaller, say, of the order of some percent of the vacuum slope, one would instead speak of a "weak reggeization". Such a "weak reggeization" may just be due to a proper treatment of the electromagnetic radiative corrections, or may correspond to a simulation of a more complex behaviour originated by electromagnetic radiative corrections.

Experimental evidence, so far would at most favor a weak reggeization rather than a strong reggeization.

The consequences of a Regge behaviour of the photon on the description of electron-proton scattering would be of various types.

The amplitude would contain six invariant functions instead of the usual two form factors depending on momentum transfer. Keeping only those amplitudes that reduce, in the limit of a photon with fixed angular momentum, to the usual form factors, one finds that the dominant modification (for large  $z$ ) is the overall factor in the amplitude  $z \alpha_1(t) - 1$  where  $z$  is the cosine of the center-of-mass angle in the crossed reaction (proton-antiproton annihilation into electrons).

To test the presence of such a factor it is preferable to do experiments at large momentum transfer, for a maximum available energy, provided  $z$  is sufficiently larger than one.

The two-photon terms, we have previously discussed, by interfering with the one-photon terms, give rise to deviations from the Rosenbluth formula that have different sign for electron and for positron scattering. The kind of deviations discussed here would instead be the same for both electron and positron scattering.

Electron-electron scattering (as obtainable with colliding beam techniques) will certainly offer a very clean test of the hypothesis of Regge-like behaviour of the photon. Here, again, the dominant feature will be, under the usual limitations, the appearance of the factor  $z \alpha_1(t) - 1$  in the amplitude.

b) The electron and the muon as Regge poles.

Electron-positron annihilation into two photons would be a possible test for a Regge behaviour of the exchanged virtual electron. Here one would have to look for a factor  $z \alpha_e(t) - 1/2$  in the amplitude, which, under the usual limitations, is the expected dominant modification arising from a Regge behaviour of the electron associated with a trajectory  $\alpha_e(t)$ .

Muon photoproduction in the field of a nucleus can be used to test a Regge-like behaviour of the virtual muon that is exchanged.

Unfortunately as pointed out by Salecker<sup>(14)</sup> the experimental data, at present, do not allow even to exclude possible slopes of the electron or of the muon trajectories of the order

of the vacuum trajectory.

c) Salecker also describes an indirect way of obtaining some information on the photon trajectory from the very accurate measurements of the muon magnetic moment. One writes down a dispersion relation for the muon magnetic form factor and makes use of unitarity to calculate the imaginary part. If one keeps only intermediate states consisting of a muon pair the imaginary part is proportional to the muon-muon scattering amplitude. For a reggeized photon such an amplitude will be modified from the perturbation theory expression and the modification will depend on the Regge trajectory of the photon. A rigorous application of this idea is made difficult from the fact that one has to integrate on the dispersion variable thus requiring a good approximation for the muon-muon amplitude on a wide range of energies. The rough evaluation by Salecker only gives an upper limit for the photon slope of the order of 10% of the slope of the vacuum trajectory.

d) Behaviour of the infrared terms.

Excluding, on experimental grounds, a "strong reggeization" of the photon the question remains whether quantum electrodynamical radiative corrections give rise to a photon trajectory.

Let us consider electron-electron scattering. It may happen that, by summing up all the diagrams corresponding to the exchange of an odd number of photons in the limit of very high energy and for fixed momentum transfer one obtains an expression of the form of that obtained by summing the contributions from different Regge trajectories.

One of these trajectories would be the photon trajectory. It will contain the photon and maybe also other poles with spin 3, 5 etc.

Similarly the exchange of an even number of photons may also lead to a Regge asymptotic behaviour or to a superposition of such behaviours. These trajectories will all have charge conjugation quantum number  $C = +1$ , whereas those arising from the exchange of an odd number of photons will have  $C = -1$ .

Levy<sup>(15)</sup> points out that the infrared part of the scattering amplitude, following the general rules given by Yennie, Frautschi and Suura<sup>(16)</sup>, can be separated according to

$$M = e^{(B+B')\frac{-}{M}} \quad (42)$$

where  $M$  does not contain any infrared contribution and  $B$  and  $B'$  are given explicitly as integrals depending on the four-momenta of the initial and final particles. Levy calculates the sum  $B+B'$  in the limit of large incident energy ( $s \rightarrow \infty$ ) and for finite momentum transfer ( $t$  fixed). The calculation shows that in this limit  $B + B' \simeq -i\alpha \ln(s/\lambda^2)$ , where  $\lambda$  is the vanishing photon mass, thus excluding in particular the possibility of Regge-like behaviour of these terms, when substituted in (42).

In fact, one should note, it would have been quite surprising and rather different from the spirit of a Regge hypothesis, if the summed-up infrared contributions would have exhibited a Regge behaviour in the high energy limit.

Indeed one should remember that in the infrared approximation one is summing up not only elastic contributions but also inelastic contributions corresponding to the emission of low energy photons that cannot be detected by the apparatus.

These inelastic contributions correspond to different physical processes (emission of one, of two, of three photons etc) and are summed up incoherently and added to the elastic contribution to give rise to the approximate expression (42). The hypothesis of a Regge behaviour corresponds instead to a particular form of the elastic amplitude alone.

e) Levy proceeds further in the examination of high energy corrections and calculates the contribution to the elastic amplitude from the exchange of two photons and finds that terms of the form  $(\ln s)^2 M_1$  and  $(\ln s) M_1$  (where  $M_1$  is the lowest order matrix element corresponding to the exchange of one photon), which could, when summed with higher order term, give rise to a behaviour like  $s^\alpha(t)$ , cancel in the high energy limit. Also a discussion (necessarily incomplete) of higher order contributions seems, according to Levy, to show no evidence for a

Regge-like behaviour of the photon.

A general investigation on the possibility of deriving Regge trajectories from field theory is being carried out by Gell-Mann, Goldberger, Low, Singh, and Zachariasen<sup>(17)</sup>. The authors have examined in particular the problem of a spinor field (nucleon) coupled to a heavy photon field. They find that the elementary spinor nucleon changes into a moving pole in the angular momentum plane after inclusion of the radiative corrections due to its coupling to the heavy photon.

In the same approximation however, a scalar nucleon remains a fixed pole in the angular momentum plane, unless, instead of the usual field theory, one uses a more complicated (presumably unrenormalizable) field theory. In any case the kind of modification would correspond to a non-minimal coupling.

### 3 - FORM FACTORS -

#### 3.1 - Dispersion theory calculations.

As well known, the experimental discovery of a large electromagnetic radius for the isovector nucleon charge distribution, led to the proposal, by Frazer and Fulco<sup>(18)</sup>, of a pion-pion p-wave resonance. The squared energy of the resonance,  $t_r$ , was found by Frazer and Fulco to lie between 10 and 16 (in units of squared pion mass), in order to fit the data with their dispersion theory calculation.

A different dispersion theory calculation by Bowcock, Cottingham and Lurie<sup>(18)</sup> led to a fit with  $t_r$  around 20.

A fit to the experimental data is provided by the following form factors

$$G_E^V(t) = e/2 \left( 1 - a_1 + \frac{a_1 t_1}{t_1 - t} \right) \quad (43)$$

$$G_M^V(t) = (1.83)e/2M \left( 1 - a_2 + \frac{a_2 t_2}{t_2 - t} \right)$$

where the parameters, according to De Vries, Hofstadter, and Herman<sup>(19)</sup>, take on the values:  $a_1 \cong 0.92$ ,  $a_2 \cong 1.15$ ,  $t_1 \cong 18$ , and  $t_2 \cong 18$ .

The form factors (43) have the form of the sum of a subtraction constant and a pole term. The pole term is supposed to arise from the pion-pion p-wave resonance.

It is also well-known that a pion-pion resonance, the  $\rho$ -meson, with a mass of 760 MeV, corresponding to  $t_r = 29$ , and a full width of  $\sim 130$  MeV, has been experimentally discovered.

It has been natural to consider the  $\rho$ -meson as responsible for the pole in (43).

However, the large difference between the experimental  $\rho$ -meson squared mass  $t_r = 29$  and the values of  $t_1$  and  $t_2$  experimentally determined,  $t_1 \cong t_2 = 18$ , may cast some doubt on the complete correctness of this attribution.

The experimental  $\rho$ -meson mass is in fact much higher than the location of the poles in the empirical fit to the form factors. The calculations by Frazer and Fulco and by Bowcock, Cottingham and Lurie, though in disagreement between each other, would in fact suggest a lower value for the resonant mass. Can a more sophisticated theory, including the present more complete information on the pion-pion scattering amplitude near the resonance, explain the pole term in (43) in terms of a pion-pion scattering dominated by the  $\rho$  resonant state?

This question has recently been answered by Ball and Wong<sup>(20)</sup>. They perform a calculation of the isovector charge and anomalous magnetic moment form factors assuming that the two-pion intermediate state determines the low energy part of the spectral function, whereas a subtraction constant represents the high energy part of the spectral function. The resulting expressions can be compared with the Clementel-Villi fits (43). The calculations are performed by using a pion-pion p-wave phase-shift adjusted from the  $\rho$  resonance, and by including the exchange of a nucleon and a  $N^*$  in the  $NN \rightarrow 2\pi$  amplitude, both represented as Regge po-



les. Two parameters are introduced at this point and they are determined from fitting the pole term in the form factors (43), at zero momentum transfer.

It turns out from Ball and Wong calculation that the effective mass of the resonant pion-pion state is shifted to a much lower energy than the  $\rho$ -meson mass. More precisely, the pole term in (43) turns out to be singular at  $t_1 \cong t_2 \cong 20$ , in spite of the fact that the pion-pion cross-section has its maximum at  $t_r = 29$ .

The two pion contribution to the isovector nucleon form factor satisfies the representation

$$G_{E, M}^{(2\pi)}(t) = \frac{1}{4} \int_4^{\infty} \frac{dt' \text{Im} G_{E, M}^{(2\pi)}(t')}{t' - t} \quad (44)$$

The calculation of the imaginary parts is performed by keeping the  $2\pi$  intermediate state. From the graph of fig. 10 we see that the imaginary parts will be of the form of products of the pion form factor  $F_{\pi}^{\kappa}(t)$  and of the appropriate combinations,  $\Gamma_{E, M}(t)$ , of the two nucleon-antinucleon scattering amplitudes with  $J=1$ ,  $P=-1$ ,  $C=-1$ . More precisely

$$\text{Im} G_{E, M}^{(2\pi)}(t) = -e(\frac{1}{4}t-1)^{3/2} t^{-1/2} F_{\pi}^{\kappa}(t) \Gamma_{E, M}(t) \quad (45)$$

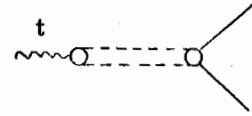


FIG. 10

The amplitudes  $\Gamma_E(t)$  and  $\Gamma_M(t)$  can be written as linear combinations of helicity amplitudes  $f_{\pm}^{(1)}(t)$  and  $f_{\mp}^{(1)}(t)$  for  $NN \rightarrow 2\pi$ . These amplitudes are taken to satisfy the dispersion relations

$$f_{\pm}^{(1)}(t) = \frac{1}{\pi} \int_{-\infty}^a \frac{\text{Im} f_{\pm}^{(1)}(t')}{t' - t} dt' + \frac{1}{\pi} \int_4^{\infty} \frac{\text{Im} f_{\pm}^{(1)}(t')}{t' - t} dt' \quad (46)$$

where the right-hand cut extends over the absorptive region for the annihilation process, whereas the right-hand cut originates from the cuts and poles in the crossed channel corresponding to  $\pi N \rightarrow \pi N$  ( $s$ -channel). Unitarity requires that the  $\Gamma(t)$  have the  $\pi$ - $\pi$  scattering phase for  $4 < t < 16$ . This condition can be assumed to hold also for higher  $t$ -values, as long as inelastic processes can be neglected. The discontinuities on the left-hand cut are related to analytical continuations of the polynomial expansions of pion-nucleon scattering. At this point; differently from Frazer and Fulco, Ball and Wong make the assumption that the dominant terms are the nucleon pole and the 3-3 resonance, and that both can be treated as Regge poles in  $s$  (Fig. 11)

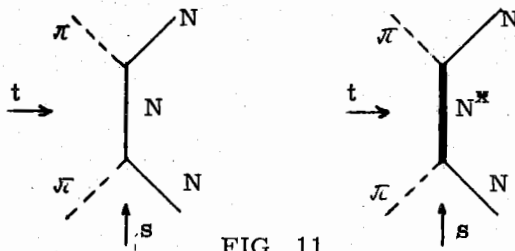


FIG. 11

The nucleon and  $N^*$  are thus both treated as pole terms multiplied by the factors  $(t/4)^{\alpha_N(s) - 1/2}$  and  $(t/4)^{\alpha_{N^*}(s) - 3/2}$  where  $\alpha_N(s)$  and  $\alpha_{N^*}(s)$  describe the nucleon and  $N^*$  trajectories respectively. Only the linear terms are kept in the expansion of  $\alpha_N$  and  $\alpha_{N^*}$  near  $m^2$  and  $m_{33}^2$ , thus introducing as parameters the two derivatives  $\alpha'_N(m^2)$  and  $\alpha'_{N^*}(m_{33}^2)$ , of which only the sign is known. These parameters are adjusted such as to fit  $G_M^{(2\pi)}(0)$  and  $G_M^{(2\pi)}(0)$ , as given by (4, 3). The pion form factor,  $F_{\pi}(t)$ , in (45), is, as usual, directly related to the pion-pion scattering amplitude in  $p$ -wave.

The shift in the effective  $\rho$ -mass is due to the combination of two effects. First, the pion form factor has its maximum at  $t = 25$  for a pion-pion cross-section with a maximum at  $t = 29$ . Second, the factor  $\Gamma(t)$  gives a larger weight to smaller values of  $t$  producing a further shift. The effective  $\rho$ -mass resulting from this calculation is around 600-650 MeV, to be compared with the observed mass of 760 MeV (used as an input in the calculation).

The main limitation of this theory lies perhaps in the assumption that only the two-pion intermediate state is important in the determination of the spectral function, apart from the added subtraction constant. The inclusion of four-pion intermediate states would be quite un-

feasible in its generality. However if the recently discovered  $\omega - \pi$  resonance (the so-called Buddha) turns out to have the appropriate quantum numbers to couple to a single photon ( $T = 1, J = 1, P = -1, C = -1$ ) an approximate treatment of the four-pion intermediate states in terms of this resonance becomes reasonable. Such calculations are in progress. The contribution from these states may turn out to be important in the time-like region of the  $p\bar{p} \rightarrow e\bar{e}$  experiment.

We shall not discuss here the isotopic scalar form factors. Their calculation appears to be substantially more difficult than for the isovector form factors. Phenomenological fits can be based on the simulation with delta functions of the contribution to the imaginary parts from the known resonances with  $T = 0, J = 1, P = -1$ , and  $C = -1$ , namely the  $\omega$  and the  $\varphi$ .

### 3.2 - Consequences of symmetries; unitary symmetry.

We shall first discuss a formal question on which not much work has yet been done, but which might eventually lead to some progress.

In his paper on "Symmetries of Baryons and Meson" Gell-Mann<sup>(21)</sup> stressed the possible relevance of equal time commutation relations between components of the currents in providing non-linear relations for matrix elements.

The simplest case is that of the isotopic spin current. It is of direct interest to us because the isovector form factors are essentially matrix elements of the isotopic spin current.

The isotopic spin components  $I_i$  obey, as well-known, the commutation relations:

$$[I_i, I_j] = ie_{ijk} I_k \quad (47)$$

where  $e_{ijk}$  is the invariant antisymmetric tensor. The local isotopic spin current has components  $j_\mu^{(i)}(x)$  which are conserved, that is

$$\frac{\partial}{\partial x_\mu} j_\mu^{(i)}(x) = 0 \quad (48)$$

and such that

$$I_i = -i \int j_4^{(i)}(x) d^3x \quad (49)$$

From (48) and (49) it follows that  $I_i$  is conserved.

The commutator

$$[j_4^{(i)}(\vec{x}, t), j_4^{(e)}(\vec{x}', t)]$$

at equal times must be zero for  $\vec{x} \neq \vec{x}'$  because of the requirement of microcausality. Assuming that is not more singular than a delta function we have, using (47) and (49),

$$[j_4^{(i)}(\vec{x}, t), j_4^{(e)}(\vec{x}', t)] = -e_{iek} j_4^{(k)}(\vec{x}, t) \delta(\vec{x} - \vec{x}'). \quad (50)$$

The equal-time commutation relation (50) leads to non-linear sum rules for matrix elements.

For instance, the pion form factor  $F(k^2)$  is related to the matrix element of the isotopic spin current taken between one-pion states. If one takes the matrix element of (50) between one-pion states with momenta  $p$  and  $p'$  one finds the relation:

$$\begin{aligned} (p_0 + q_0)(p'_0 + q_0) F_\pi(-(p-q)^2) F_\pi(-(p'-q)^2) - (p_0 - q_0)(p'_0 - q_0) F_\pi(-(p+q)^2) F_\pi(-(p'+q)^2) = \\ = 2(p_0 + p'_0) q_0 F(-(p-p')^2) \end{aligned} \quad (51)$$

where  $q$  is a four-vector such that  $q^2 = -m^2$ , and we have not written down explicitly all the inelastic terms in the sum on the left-hand-side.

From (51) one sees that, neglecting the inelastic terms,  $F(k^2)$  is unity.

It is interesting to note that the non-linear nature of the commutation relations, reflected by the non-linear character of the functional equation (51), makes it possible to fix a scale for the matrix element. (For instance (51), with inelastic terms neglected, gives  $F(k^2) = 1$ , and not simply  $F(k^2) = \text{constant}$ ).

A similar possibility is not offered by the dispersion relation for matrix elements of weak or electromagnetic currents which are linear and homogeneous: as such they may be useful in determining the momentum dependence of the form factors but they will not give their absolute values.

A classical example is that of the axial vector renormalization constant  $G_A$  which cannot be determined from the linear homogeneous dispersion relation for the axial form factor, and must be introduced as a subtraction constant.

The advantage of using equal time commutation relations, such as (50), will perhaps be most sensible for theories with broken symmetries, such as the unitary symmetric theory.

Biriz and Pietschman<sup>(22)</sup> have made use of the commutation relations among the components of the isovector currents to derive equations relating the isovector form factors and the electroproduction matrix elements. The equations are studied in the static theory where they lead to results that can be checked directly. However the application of the procedure to the relativistic theory seems rather cumbersome.

Furthermore it should be kept in mind, that in contrast to the usual application of unitarity in dispersion relations, where some intuitive justification can be given for neglecting higher inelastic terms in the unitarity series, no similar justification can be presented in this case.

### 3.3 - Consequences of unitary symmetry.

It was pointed out<sup>(23)</sup> some time ago that there exists a subgroup of the unitary symmetry group  $SU_3$  that leaves the electromagnetic current invariant. This fact was used to derive relations among electromagnetic amplitudes, some of them valid at any electromagnetic order.

There has been recently much interest in the subject and systematic treatments have appeared<sup>(24)</sup> (we shall always refer to the octet version of  $SU_3$  symmetry).

The  $SU_2$  subgroup of  $SU_3$  that leaves the electromagnetic current invariant is sometimes referred to as the U-spin group.

The  $SU_3$  group has eight generators  $F_1, F_2, \dots, F_8$ , which satisfy the commutation relations  $[F_m, F_n] = if_{mnl}F_l$ .

One can form three different sets of generators such that in each set one has an angular momentum operator and a corresponding hypercharge operator.

One set consists of the generators  $F_1, F_2, F_3$  and  $F_8$ . It is interpreted as the isospin-hypercharge set.

A second set consists of  $F_3, F_6, F_7$  and  $F_8$ . One finds that these four generators commute with the electromagnetic current. They form the set of U-spin and the associated hypercharge which coincides with electric charge apart from a sign.

There is a third set with the property of containing an angular momentum and a hypercharge operator but it is not yet clear whether it has a physical interpretation.

The use one can make of such results is pretty obvious.

Consider the usual problem of breaking  $SU_3$  invariance by introducing interactions that are not fully invariant but only leave isospin and hypercharge conserved.

On this problem one knows a great deal: one knows all the consequences of charge independence which are results valid at all orders in the symmetry breaking interaction; and one also knows results valid at the first order in the symmetry breaking interaction, at the second order in the symmetry breaking interaction etc.

A result valid at first order in the symmetry breaking interaction is Okubo's first order mass-formula, that gives the mass of the members of a multiplet as  $M = a + bY + cZ$ , where  $a$ ,  $b$  and  $c$  are constants in the multiplet,  $Y$  is the hypercharge and  $Z = (1/4)Y^2 - I(I + 1)$ . At the second order the mass formula is  $M = a + bY + cZ + dY^2 + eYZ + fZ^2$ .

Now consider the problem of breaking  $SU_3$  invariance by turning on electromagnetic interactions (as arising from the minimality rule). Other symmetry breaking effects are neglected. In the isotopic-spin problem one was left with a theory invariant only under isospin and hypercharge transformations, in this problem one is similarly left with a theory invariant only under U-spin and electric charge transformations. The two problems are mathematically equivalent.

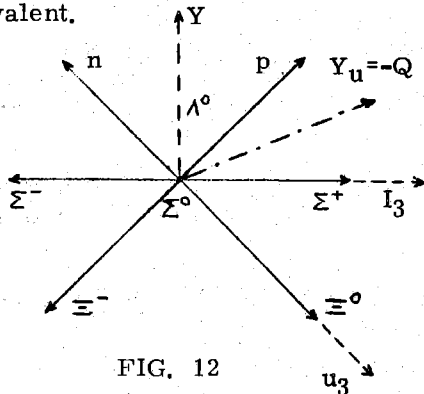


FIG. 12

One can use this equivalence to transpose directly the known results for the first problem to the second problem.

It has been pointed out by Levinson, Lipkin and Meshkov that the classification of the particles in a multiplet by means of U-spin and the associated hypercharge (which coincides with the electric charge apart from the sign) can be read off from the weight diagram for the representation by a rotation of coordinates. This is illustrated in the following graph for the baryon octet (fig. 12).

The coordinates  $Y$ ,  $I_3$  give the hypercharge and third components of isospin for the particle. The coordinates  $Y_u = -Q$  and  $U_3$  similarly give the charge (with reversed sign) and the third component of U-spin.

In a proper treatment one has to take into account the proper sign for each particle state and substitute for  $\Sigma^0$  and  $\Lambda^0$  appropriate linear combinations of them.

Consequences of charge independence that are valid to all orders can now readily be translated in relations valid at any electromagnetic order for an  $SU_3$  invariant theory broken by electromagnetism.

For instance, one finds, by directly looking at the diagram  $\langle \Sigma^+ | j \dots j | \Sigma^+ \rangle = \langle p | j \dots j | p \rangle$  where  $j$  is any product of electromagnetic current operators.

Similarly the Okubo mass formulae can be simply transposed to describe first order electromagnetic effects and second order electromagnetic effects respectively, by simply changing  $Y$  with  $-A$  and  $\vec{I}$  with  $\vec{U}$ .

The main results found by these methods had already been obtained<sup>(23)(25)</sup>. We mention for instance the relation between the  $\pi^0 \rightarrow 2\gamma$  amplitude and the  $\eta^0 \rightarrow 2\gamma$  amplitude, that, after phase-space correction, predicts a width for  $\eta^0$  of 140 eV assuming a  $\pi^0$  lifetime of  $1.5 \times 10^{-16}$  sec. Also interesting is the prediction of a  $\Lambda$  magnetic moment of one-half of the neutron magnetic moment,  $\mu_\Lambda = -0.95$  nucleon magnetons, in disagreement with one of the two existing experiments. A relation among the electromagnetic mass splittings of baryons,  $\delta M(\Xi) - \delta M(\Xi^0) = \delta M(\Sigma^-) - \delta M(\Sigma^+) + \delta M(p) - \delta M(n)$  is satisfied within experimental errors (it gives  $6.7 \pm 0.5$  MeV =  $7.4 \pm 9.2$  MeV).

#### 4 - ELECTRON SCATTERING ON DEUTERIUM $H^3$ AND $He^3$ -

##### 4.1 - Elastic electron-deuterium scattering.

###### a) Determination of the deuteron form factors.

The study of the reaction



gives direct information on the deuteron electromagnetic form factors. By additional theoretical considerations, and under some definite approximations, one may succeed in expressing

the deuteron form factors through the form factors of the nucleons. The limits of validity of the approximations used are not however theoretically clear and much caution must be used before deriving definite conclusions.

The laboratory differential cross-section of (52) can be written in the form:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[ A(k^2) + B(k^2) \text{tg}^2 \frac{\theta}{2} \right] \quad (53)$$

where  $\theta$  is the laboratory scattering angle,  $K^2$  is the square of the virtual photon four-momentum, and  $A(K^2)$  and  $B(K^2)$  are quadratic functions of the deuteron form factors.

The cross-section formula (53) is identical in its functional form to the Rosenbluth formula. In fact we have seen in section 1 that the linear  $\text{tg}^2(\theta/2)$  behaviour, implied by (53), is a direct consequence of the vector nature of the exchanged photon, and is, in particular, independent of the spin of the target.

The same linear dependence on  $\text{tg}^2(\theta/2)$  holds also for inelastic electron-deuteron scattering; however, in that case  $A$  and  $B$  are functions of  $K^2$  and of  $W$ , the total energy of the outgoing nucleons in their center-of-mass system.

In the cross-section formula (53) for the elastic process (52), the coefficients  $A(K^2)$  and  $B(K^2)$  can be expressed in terms of the deuteron form factors.

The deuteron, as any spin one particle, is described by three independent form-factors. Derivations of the relativistic elastic cross-section formula have been given by Glaser and Jaksic<sup>(26)</sup> and by Gourdin<sup>(27)</sup>.

It is convenient to introduce, in analogy to what has been done for the nucleon, a charge form factor  $F_C(K^2)$ , an electric quadrupole form factor  $F_Q(K^2)$ , and a magnetic dipole form factor  $F_M(K^2)$ .

As for the nucleon case, these form factors are such that, in the Breit system, they are the Fourier transforms of distributions that describe the charge, electric quadrupole, and magnetic dipole interactions with static electric and magnetic fields.

In terms of these form factors one has:

$$\begin{aligned} A &= F_C^2 + \frac{1}{18} \left(\frac{K^2}{M_D^2}\right)^2 F_Q^2 + \frac{1}{6} \frac{K^2}{M_D^2} \left(1 + \frac{K^2}{4M_D^2}\right) F_M^2 \\ B &= \frac{1}{3} \frac{K^2}{M_D^2} \left(1 + \frac{K^2}{4M_D^2}\right) F_M^2 \end{aligned} \quad (54)$$

where  $M_D$  is the mass of  $D$ .

The derivation of these results is fully relativistic.

Experiments of elastic electron scattering on deuteron only measure the two coefficients.  $A$  and  $B$ ; a determination of the three deuteron form factors will thus require additional experiments, for instance polarization experiments [only for small momentum transfers can one neglect the quadrupole form factor in  $A$  and solve the equations in terms of  $F_C^2$  and  $F_M^2$ ].

Gourdin and Picketty<sup>(27)</sup> point out that the difference of the cross-sections for transverse and longitudinal polarized recoil deuteron,  $\sigma_T - \sigma_L$ , gives an additional relation, allowing the determination of the three form factors.

The difference is given by

$$\sigma_T - \sigma_L = \frac{1}{3} \left[ F_C^2 - \frac{16}{3} \frac{K^2}{4M_D^2} F_C F_Q - \frac{8}{9} \left(\frac{K^2}{4M_D^2}\right)^2 F_Q^2 \right] \sigma_{\text{unpolarized}}$$

Note that the ratio between  $\sigma_T - \sigma_L$  and  $\sigma_{\text{unpolarized}}$  is independent of the magnetic form factor  $F_M$  and, as consequence of this, it is also independent of the scattering angle.

It is also interesting to remark that, if one knows all the three deuteron form factors, one has in principle the possibility of testing directly the validity of any theory (based on impulse approximation) that gives the deuteron form factors in terms of the two isoscalar nucleon form factors. Deviations would indicate a breakdown of the non-relativistic impulse approximation.

The impulse approximation allows one to relate the deuteron form factors to the nucleon form factors.

b) Impulse approximation for the deuteron form-factors.

The deuteron current, in the impulse approximation, is written as a sum of the neutron and of the proton currents

$$j_{\mu}^{(D)} = j_{\mu}^{(p)} + j_{\mu}^{(n)} \quad (55)$$

One sees from (55) that only the isoscalar nucleon form factors  $G_E^{(s)}$  and  $G_M^{(s)}$  will appear in the final result.

In the impulse approximation one finds<sup>(27)</sup>

$$F_C = G_E^{(s)} C_E, \quad F_Q = G_E^{(s)} C_Q, \quad F_M = \frac{M_D}{M} \left[ G_M^{(s)} C_S + \frac{1}{2} G_C^{(s)} C_L \right] \quad (56)$$

where  $C_E$ ,  $C_Q$ ,  $C_S$  and  $C_L$  are functions of  $K^2$  and depend on the deuteron wavefunction.

The deuteron wave-function has the form  $u(r) + S_{12} w(r)$  where  $S_{12}$  is the tensor operator. The structure functions  $C_E$ ,  $C_Q$ ,  $C_S$  and  $C_L$  can be expressed as linear combinations of the following integrals

$$\begin{aligned} u_0 &= \int_0^{\infty} u^2(r) j_0\left(\frac{Kr}{2}\right) dr, & w_0 &= \int_0^{\infty} w^2(r) j_0\left(\frac{Kr}{2}\right) dr, \\ u_w &= \int_0^{\infty} u(r) w(r) j_2\left(\frac{Kr}{2}\right) dr, & w_2 &= \int_0^{\infty} w^2(r) j_2\left(\frac{Kr}{2}\right) dr \end{aligned} \quad (57)$$

One has:

$$\begin{aligned} C_E &= u_0 + w_0, & C_Q &= 6\sqrt{2} K^{-2} M_D^2 \left( u_w - \frac{1}{2\sqrt{2}} w_2 \right), \\ C_S &= u_0 - \frac{1}{2} w_0 - \sqrt{2} u_w - w_2, & C_L &= \frac{3}{2} (w_0 + w_2) \end{aligned}$$

In (57)  $j_0$  and  $j_2$  are the spherical Bessel functions of order 0 and 2.

After substitution of (56) into (54) A and B become quadratic functions of  $G_E^{(s)}$  and  $G_M^{(s)}$ . One can invert the resulting expressions obtaining  $G_E^{(s)}$  and  $G_M^{(s)}$  in terms of the measured coefficients A and B.

As one has to invert quadratic equations, there are sign ambiguities in the equations giving the G's in terms of A and B.

c) The calculation of the structure functions depends on the choice of the deuteron wave-function. Bialkowski<sup>(28)</sup> makes use of rather refined expressions for the deuteron wave-function. He obtains  $G_E^{(s)}$  and  $G_M^{(s)}$  for the nucleon after substituting (56) into (54), inverting the resulting equations, and inserting for A and B the experimental values found at Orsay<sup>(29)</sup>. Making use of the Drickey and Hand data for the proton form factor<sup>(30)</sup> he obtains the neutron form factors  $G_E^{(N)}$  and  $G_M^{(N)}$ .

The sign ambiguities, after inverting (54), can be solved by physical considerations: a negative sign in front of the expression for  $G_E^{(s)}$  would lead to a negative value for such a form factor (which should have instead for the considered momenta the same sign of the proton char-

ge form factor); similarly a sign ambiguity in  $G_M^{(s)}$  can be solved by excluding one choice which would lead to too high values for  $G_M^{(N)}$ .

d) Relativistic effects.

The derivation of Eqs. (56) and (57) is completely non-relativistic. Relativistic effects have been studied by Tran Thanh Van<sup>(31)</sup> in the impulse approximations. In practical application the results do not significantly differ from those of Bialkowski.

In the absence of a rigorous relativistic theory for a bound system like the deuteron the calculation of relativistic effects must be done under approximations whose range of validity is rather hard to establish. The typical impulse approximation graph of Fig. 13 is calculated

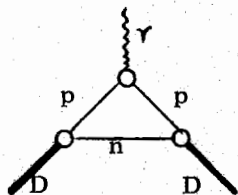


FIG. 13

under various approximations: (1) the form factors at the photon vertex are taken for nucleons on the mass-shell; (2) the free nucleon propagators are inserted for the internal nucleon lines; (3) the vertex  $D \rightarrow n + p$  is derived by using a Bethe-Salpeter equation with separable potential.

e) Possible breakdown of the impulse approximation.

Apart from the possible errors in the evaluation of relativistic effects, a source of errors may be the use of the impulse approximation.

The simplest exchange current contribution (Fig. 14) can however be shown not to contribute: the photon must be isoscalar ( $\gamma + D \rightarrow D$  is a zero-zero transition), therefore cannot couple to two pions.

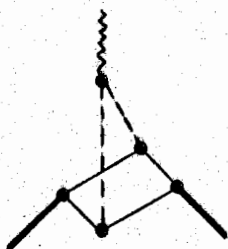


FIG. 14

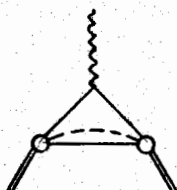


FIG. 15

A similar graph with three pions (two coupled to one nucleon, one the other nucleon) is however allowed-although perhaps dominated by the "impulse approximation" graph where the three pion couple to the same nucleon.

However a graph like, for instance, is not contained in the impulse approximation, and its contribution is not easy to evaluate (Fig. 15).

As we have already mentioned, polarization measurements can be used to test empirically the validity of the impulse approximation in the description of the deuteron form-factors.

#### 4.2 - Inelastic electron-deuterium scattering.

There has not been much progress lately in the theory of inelastic electron-deuteron scattering. For completeness we shall here review the main points also for this problem, the experimental interest of which is connected to the possibility of determining the neutron form factors.

The experiments on slow neutron scattering by atoms, first started by Fermi and Marshall, are indeed only able to give the mean square radius of the neutron charge distribution.

The simplest measurement in the inelastic process

$$e + d \rightarrow e + n + p \quad (58)$$

is the observation of the final scattered electron at an angle  $\theta$  and with final energy  $E'$ . For fixed  $\theta$  the cross-section shows a peak around a definite value of  $E'$ . The main contribution to the peak comes from events where the electron has been scattered from a single nucleon in the deuteron (proton or neutron), behaving approximately as free. Using such a description, the peak cross-section is proportional to the sum of the proton and of the neutron cross-section. Thus, a measurement of the cross-section at the peak, after subtracting the proton contribution, will give the electron cross-section on a neutron. However the neutron contribution is much smaller than the proton contribution, making the analysis difficult. Furthermore there are theoretical uncertainties, coming from: (i) the final state interaction of  $n$  and  $p$ ; (ii) the deuteron wave-function, Durand<sup>(32)</sup>, extending the calculations by Jankus<sup>(33)</sup>, has given an appro

ximate evaluation of the effect of final state interaction. With the Signell-Marshak phase shifts he finds a lowering of the peak height of about 5% and a corresponding largening of the peak. The uncertainties from the deuteron wave-function on the peak cross-section are estimated to be less than  $\sim 5\%$ . Durand has given the formula:

$$\frac{d^2\sigma_D}{d\Omega dE'} = 4.57 \times 10^{-3} \text{ MeV}^{-1} (1 \pm 0.05) \frac{4M^2}{KW} \frac{E}{E'} \left( \frac{d\sigma_p}{d\Omega} + \frac{d\sigma_n}{d\Omega} \right) \quad (59)$$

where  $E$  and  $E'$  are the initial and final electron energy,  $K$  is the transferred four-momentum, and  $W$  the total energy of the neutron-proton system in its center of mass.

A much better determination of the neutron form factors from inelastic electron scattering on deuterium can be made in experiments where one of the recoil nucleons is observed. One selects the events with the recoil neutron momentum in a narrow cone around  $\vec{K}$  (momentum transferred to the nucleons in the laboratory system) and the events with the recoil proton in the same cone. The ratio between the number of events with the neutron in the cone and the number of events with the proton in the cone is essentially the ratio of the neutron to the proton differential cross-section. Experimentally it is more convenient to select the events with the recoil proton in a backward cone (with respect to  $K$ ) instead of the events with the recoil neutron in the forward cone. The equality of the above ratio to the ratio of the differential cross-sections is disturbed by interference terms arising from contributions to the scattered amplitude by the spectator particle. Also final state interaction effects must be evaluated<sup>(32)</sup>. By measuring the ratio between the number of events with the neutron in the cone around  $\vec{K}$  and the number of events with the proton in the cone, one is approximately determining the ratio between the contributions of the two poles in the amplitude represented by the graphs of Fig. 16.

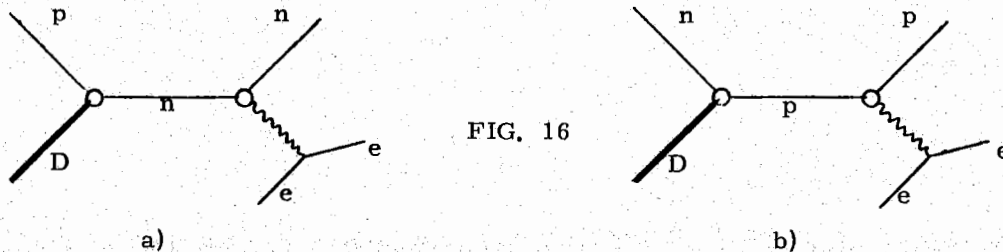


FIG. 16

The pole in diagram (a) corresponds to a recoil proton kinetic energy in the laboratory system of  $-1.1$  MeV. If  $\alpha$  is defined as the angle between the transferred momentum  $K$  and the neutron momentum, the neutron pole (diagram a) lies at  $\cos \alpha > 1$ , while the proton pole lies at  $\cos \alpha < -1$ . The selection of neutrons in forward cone around  $K$  selects a region dominated from the neutron pole; the selection of neutrons in the backward cone selects a region dominated by the proton pole. The contributions from the neutron pole and from the proton pole are proportional to the cross-sections on neutron and proton respectively. The common multiplicative constant (dependent on the deuteron structure) cancels in the ratio.

#### 4.3 - Electron scattering in $H^3$ and $He^3$ .

Elastic electron scattering in  $H^3$  and  $He^3$  has recently been shown by Schiff, Collard, Hofstadter, Johansson and Yearian to be very valuable in providing information on the neutron charge form-factor<sup>(34)</sup>. Each of the two nuclei is described by an electric charge and a magnetic moment form factor. These form factors are related to the charge and magnetic moment form factors of the proton and of the neutron, and to three structure functions of the nuclei.

Schiff gives the following expressions for the  $He^3$  and the  $He^3$  form factors

$$2F_{ch}(He^3) = 2F_L F_{ch}^{(p)} + F_O F_{ch}^{(n)} \quad (60)$$

$$\mu(He^3) F_{mag}(He^3) = \mu(n) F_O F_{mag}^{(n)} + \frac{2}{3} \mu(p) (F_O - F_L) F_{mag}^{(p)} + [\mu(He^3) - \mu(n)] F_x$$



$$F_{ch}(H^3) = 2F_L F_{ch}^{(n)} + F_o F_{ch}^{(p)} \quad (60)$$

$$\mu(H^3)F_{mag}(H^3) = \mu(p)F_o F_{mag}^{(p)} + \frac{2}{3}\mu(n)(F_o - F_L)F_{mag}^{(n)} + [\mu(H^3) - \mu(p)]F_x$$

The constants  $\mu(He^3)$  and  $\mu(H^3)$ , and similarly  $\mu(p)$  and  $\mu(n)$  are the static values of the magnetic moments in nuclear magnetons.

The structure function  $F_L$  can be thought of as the form factor describing the distribution of the centers of the like pair of nucleons, namely, the two protons in  $He^3$  and the two neutrons in  $H^3$ .

The function  $F_o$  describes instead the odd nucleon.

The form factor  $F_x$  is an exchange form factor.

The ground state wave-function is usually taken as a superposition of a space symmetric  $2S_{1/2}$ , a mixed symmetry  $2S_{1/2}$ , and three  $4D_{1/2}$  states. However the dominant state is the completely symmetric  $2S_{1/2}$ . The analysis is thus limited to this state and to the interferences with the  $2S_{1/2}$  state of mixed symmetry and with the  $4D_{1/2}$  states.

The S-D interference contributes only to the moment form factors and, for the purpose of the analysis, is included into the exchange term.

This hypothesis may be slightly in contrast with the assumption that the exchange term is the same for  $H^3$  and  $He^3$ .

Also the Coulomb repulsion in  $He^3$  is neglected.

Schiff has performed a calculation of  $F_L$  and  $F_o$  under three different assumptions for the wavefunction.

In each case there are two parameters in the problem, one measuring a size, and the other giving the amount of superposition of the S-state of mixed symmetry to the fully symmetric S-state. Information on the sign of the second parameter can be obtained from the spin-dependent part of the nucleon-nucleon force.

The analysis of the data is done using Eqs. (60) considered as four equations in the four unknowns  $F_L$ ,  $F_o$ ,  $F_x$  and  $F_{ch}^{(n)}$ . The form factors  $F_{ch}^{(p)}$ ,  $F_{mag}^{(p)}$  and  $F_{mag}^{(n)}$  are sufficiently known for the momentum transfers relevant to the experiment.

There are two sets of solutions for the four unknowns but only one of the two sets is acceptable on physical grounds.

Comparison with the data has shown good agreement with the calculated  $F_L$  and  $F_o$  with a preference for a dependence of the S-wavefunction on the internucleon distances given by an exponential of the square root of the sum of the squares of the distances. The sign of the mixed-symmetry S-wave also comes out in agreement with the prediction. The size parameters are compatible with the values derived from the  $He^3$  Coulomb energy. Schiff also determines the probability of the S-state of mixed symmetry which comes out to be of the order of 4%.

Concluding this section we would like to recall of a proposal by Drell and Zachariasen<sup>(35)</sup> for measuring the neutron form factors from inelastic electron scattering in deuterium according to the reactions



The method is a convenient variant of the extrapolation method we have described for the deuterium. In reaction (61) one extrapolates to the pole corresponding to the graph of Fig. 17 whereas in reaction (62) one extrapolates to the pole of the graph of Fig. 18.

For both extrapolations one will detect low energy deuterons with the same experimental set-up, by only changing the target. Furthermore the poles corresponding to the intermedi-

ate D for both (61) and (62) should give negligible contributions making the extrapolation procedure perhaps safer than in the case of the deuteron.

## 5 - PHOTOPRODUCTION -

### a) Phenomenological analysis.

We shall first summarize a few general notions on single pion photoproduction before describing the recent work on the subject.

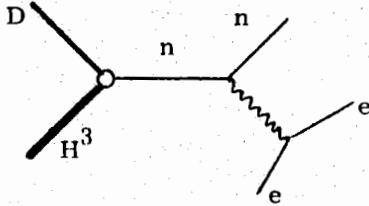


FIG. 17

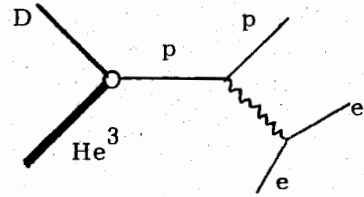


FIG. 18

We call  $s_1 = -(k+p_1)^2$ ,  $s_2 = -(k-p_2)^2$ ,  $t = -(p_1-p_2)^2$  the kinematic invariants for the single pion photoproduction process (Fig. 19).

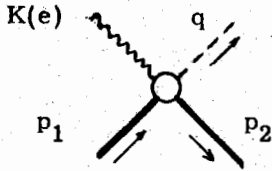


FIG. 19

The initial and final nucleon momenta are  $p_1$  and  $p_2$ , the photon momentum is  $k$ ,  $e$  is its polarization, and  $q$  is the pion momentum.

There are three independent isospin combinations:  $\tau_\alpha$ ,  $\delta_{3\alpha}$ , and  $1/2 (\tau_\alpha, \tau_3)$  where  $\alpha$  is the isospin index of the pion. The amplitude is then written as:

$$\mathcal{F} = \mathcal{F}^{(+)} \delta_{3\alpha} + \frac{1}{2} (\tau_\alpha, \tau_3) \mathcal{F}^{(-)} + \tau_\alpha \mathcal{F}^{(0)} \quad (63)$$

where  $\mathcal{F}^{(+)}$  and  $\mathcal{F}^{(-)}$  are isovector amplitudes ( $\Delta \vec{I} = 1$ ), whereas  $\mathcal{F}^{(0)}$  is the isoscalar amplitude ( $\Delta \vec{I} = 0$ ).

The  $\mathcal{F}$ 's can be further decomposed as linear combinations of the four Lorentz-invariant and gauge invariant amplitudes<sup>(36)</sup>:

$$\begin{cases} M_A = i\gamma_5(\gamma e)(\gamma K) \\ M_B = 2i\gamma_5 [(\bar{P}e)(qK) - (PK)(\gamma e)] \\ M_C = \gamma_5 [(\bar{\gamma}e)(qK) - (\gamma K)(qe)] \\ M_D = 2\gamma_5 [(\bar{\gamma}e)(PK) - (\gamma K)(Pe) - iM(\gamma e)(\gamma K)] \end{cases} \quad (64)$$

where  $P = 1/2(p_1 + p_2)$  and  $M$  is the nucleon mass.

The amplitude for  $\gamma + p \rightarrow p + \pi^0$  is  $(\mathcal{F}^+ + \mathcal{F}^0)$ ; that for  $\gamma + p \rightarrow n + \pi^+$  is  $1/\sqrt{2}(\mathcal{F}^- + \mathcal{F}^0)$ ; that for  $\gamma + n \rightarrow p + \pi^-$  is  $1/\sqrt{2}(\mathcal{F}^- - \mathcal{F}^0)$ ; and that for  $\gamma + n \rightarrow n + \pi^0$  is  $(\mathcal{F}^+ - \mathcal{F}^0)$ .

In the following table we list the multipole transitions leading to a final pion-nucleon state with relative 1 up to 4. The notation usually employed to denote the multipoles is the following:  $E^\pm$  denotes an electric multipole transition leading to a final state where  $J = 1 \pm 1/2$ ; similarly,  $M^\pm$  denotes a magnetic multipole transition leading to a final state with  $J = 1 \pm 1/2$ . We write in the table the same transitions also in the more conventional notation:  $\mathcal{E}_1(J)$  for an electric 1-pole leading to a state of total angular momentum  $J$ , and, similarly,  $M_1(J)$  for magnetic 1-pole leading to a state with total angular momentum  $J$ . We also indicate the known pion-nucleon resonances with the quantum numbers of the final state. They are called  $N$  for isospin 1/2 and  $\Delta$  for isospin 3/2; their masses in MeV are written beside their symbols; also indicated is the laboratory energy of the  $\gamma$  at resonance. The resonance at  $E_\gamma = 1300$  could possibly be  $f_{7/2}$  or  $g_{7/2}$ .

The excitation of  $N^*$  is known to be due mainly due to  $M_1^+$ ; also various analyses indicate that the transition to  $N^{**}$  is mainly due to  $E_2^-$ .

TABLE I - Multipole transitions and pion-nucleon resonances

final l	final state	multipoles	radiation multipoles	resonances
l = 0	$s_{1/2}^-$	$E_0^+$	$\mathcal{E}_1(1/2)$	
l = 1	$\left\{ \begin{array}{l} p_{1/2}^+ \\ p_{3/2}^+ \end{array} \right.$	$M_1^-$	$\mathcal{E}_1(1/2)$	N(938), nucleon; $E_\gamma < 0$
		$\boxed{E_1^+} \quad \boxed{M_1^+}$	$\mathcal{E}_2(3/2) \quad \boxed{M_1(3/2)}$	$\Delta(1238), N^*$ ; $E_\gamma = 340$ MeV
l = 2	$\left\{ \begin{array}{l} d_{3/2}^- \\ d_{5/2}^- \end{array} \right.$	$\boxed{E_2^-} \quad M_2^-$	$\boxed{\mathcal{E}_1(3/2)} \quad m_2(3/2)$	N(1510), $N^{**}$ ; $E_\gamma = 750$ MeV
		$E_2^+ \quad M_2^+$	$\mathcal{E}_3(5/2) \quad m_2(5/2)$	
l = 3	$\left\{ \begin{array}{l} f_{5/2}^+ \\ f_{7/2}^+ \end{array} \right.$	$E_3^- \quad M_3^-$	$\mathcal{E}_2(5/2) \quad m_3(5/2)$	N(1688), $N^{***}$ ; $E_\gamma = 1020$ MeV
		$E_3^+ \quad M_3^+$	$\mathcal{E}_4(7/2) \quad m_3(7/2)$	
l = 4	$\left\{ \begin{array}{l} G_{7/2}^- \\ \dots \end{array} \right.$	$E_4^- \quad M_4^-$	$\mathcal{E}_3(7/2) \quad m_4(7/2)$	} ? $E_\gamma = 1300$ MeV
		.....	.....	

We have stressed this privileged role of  $M_1^+$  and  $E_2^-$  by putting them into squares.

No similar conclusion has been reached on the excitation of  $N^{***}$ .

In a simple-minded low-energy approach to photoproduction (like that by Gourdin and Salin) one would consider the Born terms (direct, crossed, and photoelectric) (Fig. 20).

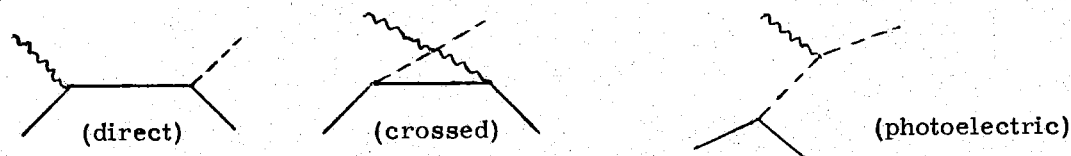


FIG. 20

The contributions from the pion-nucleon resonances (Fig. 21) and the contributions from pionic resonances (Fig. 22).

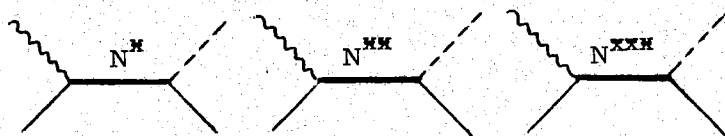


FIG. 21

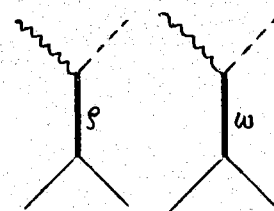


FIG. 22

We shall also refer to the direct and crossed Born terms as to the nucleon pole contributions, and to the photoelectric term as to pion pole contribution. These terms are indeed the pole terms of dispersion theory; they are calculated using the renormalized coupling constant and including the electromagnetic interaction of the anomalous moments of the nucleons.

A simple examination of the isotopic spin properties of each contribution shows that: the nucleon contribution and the contributions from nucleon isobars with  $I = 1/2$  (all indicated with N) appear in all three amplitudes  $\mathcal{Z}^{(+)}$ ,  $\mathcal{Z}^{(-)}$  and  $\mathcal{Z}^{(0)}$ ; the contributions from nucleon isobars with  $I = 3/2$  (all indicated with  $\Delta$ ) appear only in the isovector amplitudes  $\mathcal{Z}^{(+)}$  and  $\mathcal{Z}^{(-)}$  (the photon has to transmit  $\Delta I = 1$  to produce the  $N \rightarrow \Delta$  transition); the  $\omega$  only contributes to  $\mathcal{Z}^{(+)}$ , the  $\pi$  only to  $\mathcal{Z}^{(-)}$ , and the  $\rho$  only to  $\mathcal{Z}^{(0)}$  (to see this simply: in (63)  $\mathcal{Z}^{(+)}$  only contributes to  $\pi^0$  production (the index  $\alpha = 3$ ),  $\mathcal{Z}^{(-)}$  cannot contribute to  $\pi^0$  production, because of the vanishing of the commutator;  $\mathcal{Z}^{(+)}$  and  $\mathcal{Z}^{(-)}$  are isovector while  $\mathcal{Z}^{(0)}$  is isoscalar; the  $\gamma$  coupled to  $\pi + \omega$  or  $2\pi$  must be isovector, coupled to  $\rho + \pi$  must be isoscalar;  $\omega$  can

obviously only contribute to  $\pi^0$  production, whereas  $\pi^\pm$  cannot contribute to  $\pi^0$  production). In summary we have:

amplitude	contributions from
$\mathcal{F}^{(+)}$	N, $\Delta$ , $\omega$
$\mathcal{F}^{(-)}$	N, $\Delta$ , $\pi$
$\mathcal{F}^{(0)}$	N, $\rho$

We see from the table that the 3-3 resonance does not contribute to  $\mathcal{F}^{(0)}$ . This amplitude may thus be sensitive to the  $\rho$ . An experimental quantity particularly sensitive to  $\mathcal{F}^{(0)}$  is the ratio  $\pi^-/\pi^+$  (ratio of the cross-sections of  $\gamma n \rightarrow p\pi^-$  and of  $\gamma p \rightarrow n\pi^+$ ), called R, and given by  $|\mathcal{F}^{(-)} - \mathcal{F}^{(0)}|^2 / |\mathcal{F}^{(-)} + \mathcal{F}^{(0)}|^2$ .

b) The isobaric model.

Gourdin and Salin<sup>(37)</sup> use higher spin fields to describe the pion nucleon resonances and the pionic resonances. The results have a form similar to those of dispersion theory calculations apart from subtraction constants.

The crossed terms in the nucleon isobar graphs are not included. On the basis of an analysis by Dennery<sup>(38)</sup> they are assumed to be negligible.

The contribution from the 3-3 resonance,  $N^{**}$ , is calculated by describing  $N^{**}$  with a spin 3/2 field  $\psi_\mu$  ( $\mu$  is a tensor index,  $\psi$  has also spinor indices). The  $\pi NN^{**}$  vertex is described by

$$H_{\pi NN^{**}} = \frac{\lambda_1}{\mu} \bar{\psi} \psi_\mu \partial_\mu \psi + \text{h. c.} \quad (65)$$

Three gauge invariant couplings can be written down for the  $\gamma NN^{**}$  vertex

$$H^{(3)} = -e \frac{C_3}{\mu} \bar{\psi} \gamma_\mu \gamma_5 \psi_\nu + \bar{\psi}_\nu \gamma_\mu \gamma_5 \psi \bar{F}_{\mu\nu} \quad (66)$$

$$H^{(4)} = -e \frac{C_4}{\mu^2} \bar{\psi} \gamma_5 \partial_\mu \psi_\nu + \partial_\mu \bar{\psi}_\nu \gamma_5 \psi \bar{F}_{\mu\nu} \quad (67)$$

$$H^{(5)} = -e \frac{C_5}{\mu^2} \bar{\psi} \partial_\mu \gamma_5 \psi_\nu + \bar{\psi}_\nu \gamma_5 \partial_\mu \psi \bar{F}_{\mu\nu} \quad (68)$$

One can readily show that  $H^{(4)}$  and  $H^{(5)}$  are proportional in photoproduction (real photon). In fact the difference  $C_4 H_5 - C_5 H_4$  is proportional to  $(\partial/\partial X_\mu) F_{\mu\nu}$ . One chooses  $C_4 = C_5$  and has only two independent hamiltonians. The fit to the data gives  $C_3 = 0.37$  and  $C_4 = C_5 = 0.004$ .

One can conclude that the effect of the 3-3 resonance can be described only by  $H^{(3)}$ , which corresponds essentially to a magnetic dipole transition  $M_1^+$ .

The covariant form of  $H^{(3)}$  induces however some relativistic effects which give rise to a small electric quadrupole transition  $E_1^+$ .

Phenomenological analysis gives  $E_1^+/M_1^+ \cong -4.5\%$  in good agreement with such a scheme.

The contribution from the second resonance,  $N^{***}$ , can be derived by considering couplings analogous to (65), (66), (67) and (68), except that a  $\gamma_5$  must be inserted between  $\psi$  and  $\psi_\mu$  ( $N^{***}$  has parity opposite to that of  $N^{**}$ ).

The fit to the data shows that the effect of the second resonance  $N^{***}$  can be described by the hamiltonian analogous to  $H^{(4)}$  which gives essentially an electric transition.

Phenomenological analysis give  $M_2^-/E_2^- \cong 2.5\%$ .

Contributions from  $N^{***}$  are also added, in the Gourdin-Salin analysis, by assuming Breit-Wigner expressions for the relevant multipoles  $E_3^-$  and  $M_3^-$ .

The  $\rho$  meson contribution is calculated assuming a  $\gamma\pi\rho$  vertex of the form

$$\lambda \delta_{\alpha\beta} \epsilon_{\mu\nu\rho\sigma} e_\mu K_\nu q_\rho \eta_\sigma \quad (69)$$

where  $\lambda$  is a constant,  $\alpha, \beta$  are isospin indices,  $e$  is the photon polarization vector,  $K$  is the photon momentum,  $q$  is the pion momentum and  $\eta$  is the  $\rho$  polarization vector. A  $\rho NN$  vertex

is introduced of the form, in momentum space,

$$\bar{C}_1 \gamma_\tau + \frac{i}{2} C_2 \bar{\gamma}_\tau \cdot \gamma_\lambda \bar{(p_2 - p_1)_\lambda} \bar{\gamma}_\beta \quad (70)$$

where  $C_1$  and  $C_2$  are constants, and  $p_1$ ,  $p_2$  are the initial and final nucleon momenta.

To the pole contribution a subtraction term is added, as suggested by the dispersion relation approach.

Couplings similar to (69) and (70), with obvious modifications, are also assumed for the  $\omega$ . The constants  $C_1$  and  $C_2$  appearing in (70) are evidently related to the residu of the pole of  $\rho$  in the Dirac and Pauli nucleon form factors, as calculated by the graph of Fig. 23. Precisely one has:  $\text{Im}F_1^{(v)}(t) = -(e/2)\pi$  at  $t = \delta(t-t_r)$  and  $\text{Im}F_2^{(v)}(t) = -(eg/2M)\pi b \delta(t-t_r)$ , and  $a$  and  $b$  are related to  $C_1$  and  $C_2$ . One can thus obtain the ratio  $C_1/C_2$  from the nucleon form factors.

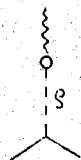


FIG. 23

One could use a similar procedure for  $\omega$ , excepts that the uncertainties are much larger.

One sees from the above summary that a large number of parameters are introduced in the Gourdin-Salin model to get a fit to the data. A fit at high energies also shows the necessity of introducing non-resonant P waves, thus increasing the number of parameters.

An interesting conclusion concerns the  $\rho$  and  $\omega$  retardation contributions: no evidence is found for such contributions. Moravicsik<sup>(39)</sup> had obtained a similar conclusion by extrapolating the data for neutral photoproduction.

The photoelectric term is found to be responsible for the small angle behaviour of the  $\pi^+$  distribution, and for the rapid decrease of the total cross-section after the 3-3 resonance.

On the other hand the crossed Born term only seems to appear in the  $\pi^0$  data after 900 MeV.

Non resonating S-waves are important in the region of the 3-3 resonance.

A satisfactory fit is obtained up to 900 MeV. Interferences between resonating multipoles and the other terms are essential for the fit.

### c) Application of dispersion theory.

This approach to photoproduction, consisting in a simple model with parameters fixed to fit the data, is very different in spirit to that by Höhler and Schmidt<sup>(40)</sup> who try to get from theory the maximum of information and look for deviations when comparing with experiments.

Höhler takes into account only the well known pole terms and the contribution from the resonant magnetic dipole. The contributions of the  $\text{Im} M_{33}$  to the dispersion integrals are calculated using for  $M_{33}$  the formula given by CGLN<sup>(36)</sup>.

$$M_{33} = \frac{\mu}{f} \frac{K}{q^2} \sin \alpha_{33} e^{i\alpha_{33}} \quad (71)$$

where, as before,  $K$  is the photon momentum and  $q$  is the pion momentum. The pion-nucleon phase  $\alpha_{33}$  is taken from the phase shift analysis directly.

This work is closely related to a previous work by Ball<sup>(40)</sup>.

In spite of the absence of adjustable parameters no remarkable differences are found when comparing with the data up to about 500 MeV.

Differences, if they will become evident, may be due to various possible sources of errors in the calculation. The  $M_{33}$  formula (71) may not hold at higher energies; final state interaction in other states may be important; the  $\rho$  and  $\omega$  contributions may become noticeable; the contributions from  $N^{*x}$  may become large.

It may be mentioned at this point that a due to understanding the lack of contributions from  $\rho$  exchange in photoproduction may be the hypothesis recently put forward by Low<sup>(41)</sup> of

an approximate conservation law that forbids  $\rho \rightarrow \pi + \gamma$ .

As far as  $N^{xx}$  is concerned, Höhler and Dietz, show, by an interference argument that it is mainly due to  $E_2^-$  (as was suggested by Peierls, and in agreement with Gourdin and Salin<sup>(37)</sup>).

d) The comparison of the Höhler and Dietz work to a recent work by McKinley<sup>(42)</sup> is very interesting. McKinley has found that the addition of other contributions to the amplitude as given by the sum of the Born terms and the contributions from the resonant multipole, appreciably modifies the result, if such terms are added following the prescriptions of CGLN.

To reduce such modifications one has to add explicitly the  $\rho$  meson term and a term corresponding to exchange of  $3\pi$  characterized by a  $\gamma - 3\pi$  coupling constant.

However the  $\gamma - 3\pi$  coupling is expected, as shown by Ball<sup>(40)</sup>, to influence mostly the multipoles  $E_0^+$  and  $M_1^-$ ; but final state interactions, different from the  $3-3$  resonant interaction, may produce effects of the same order in exactly the same multipoles.

The isoscalar subtraction constant, which was found to be large by Ball<sup>(40)</sup>, is neglected by McKinley. One may also suspect that the calculations of the additional terms performed by McKinley may be unreliable partly because of failures of the CGLN approximation and partly because of uncertainties in the  $T = 1/2$   $\pi$ -N phase shifts. A definite judgement on these questions is very hard.

One can learn from the preceding discussion that the recognition of effects such as the  $\rho$  and  $\omega$  terms and final state interaction in non- $3-3$  states, and other similar contributions, from the analysis of photoproduction data below 500-700 MeV is not at all an easy task. It may happen that the amplitude is really due, almost entirely, to the Born terms and the contributions from the resonant multipole to the dispersion integrals. Or it may happen that other contributions are singly important, but when are all added up in the squared amplitude, averaged over polarizations, their effects become masked.

We hope that the experiments with polarized photons that have recently been started at different laboratories will bring us a better understanding of such problems.

## 6 - ELECTROPRODUCTION -

### a) The pion electroproduction reactions



offer, in principle, a possibility for determining the pion form factor and the neutron form factors. At present, a determination of the pion form factor from electroproduction does not seem to be experimentally easy; on the other hand a determination of the neutron form factors from electroproduction is possible.

The electroproduction matrix element is related to the matrix element for photoproduction from a virtual photon, as shows the graph of Fig. 24.

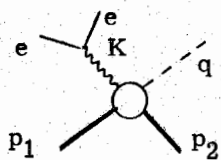


FIG. 24

One is particularly interested in the pole contributions to the reactions, which for  $\pi^+$  electroproduction (reaction 73) correspond to the graphs of Fig. 25, while for  $\pi^0$  photoproduction (reaction (72)) correspond to the graphs of Fig. 26.

A study of the feasibility of the extrapolation to the pole corresponding to the graph of Fig. 25c) was made by Frazer<sup>(43)</sup> who found that an improvement of the experimental accuracy was necessary to make the extrapolation possible.

The  $\pi^0$  electroproduction process has poles corresponding to an intermediate proton, as represented by the graphs of Fig. 26 a, b, whose residui depend on the proton form factor at  $K^2$  ( $K$  is the momentum of the virtual photon).

The possibility of a measurement of the neutron form factor from  $\pi^+$  electroproduction

can be seen from the occurrence of the graph of Fig. 25 b, with a residuum at the pole depending on the neutron form factor.

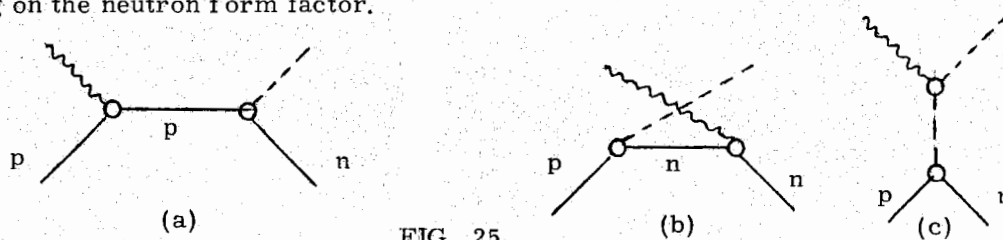


FIG. 25

Furthermore in  $\pi^+$  electroproduction the graph of Fig. 25b, interferes coherently with other graphs, such as those of Fig. 25a and 25c, thus allowing a determination of the sign of the neutron form factor.

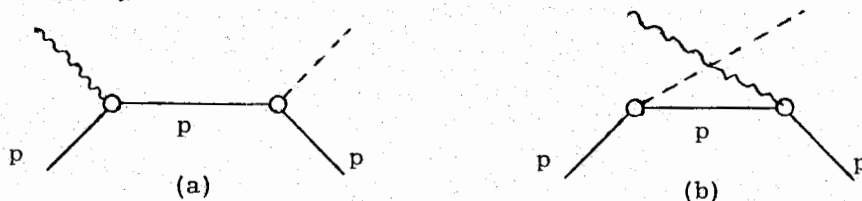


FIG. 26

In the low energy region, for fixed  $K^2$ , the cross-section will show a peak when the final pion-nucleon energy, in their center of mass, comes close to the 3-3 resonance. The dominant amplitude is the isovector magnetic dipole  $M_1^+$ , from analogy from photoproduction by a real photon, and will be proportional to the magnetic isovector nucleon form factor, that is to  $[\bar{F}_1^{(p)}(K^2) - F_2^{(n)}(K^2)]$ .

b) A recent theoretical contribution to electroproduction is due to Salin<sup>(44)</sup>, who has extended to virtual photons the model previously employed for the study of photoproduction<sup>(37)</sup>. The model consists of the Born terms plus the 3-3 resonance contribution described by the direct graph with an intermediate  $N^*$  with the coupling terms (66), (67) and (68). The  $K^2$  dependence of the  $\gamma NN^*$  vertex is deduced from the dispersion theory considerations by Denner<sup>(38)</sup> showing a proportionality to the magnetic isovector form factor. The electroproduction differential cross-section, for non-observed final pion is compared with the data of Ohlsen<sup>(45)</sup> and Hand<sup>(46)</sup> showing that the Dirac neutron form factor has perhaps a negative sign and at same time that it is practically impossible, at this moment, to extract the pion form factor from the data. The conclusions depend on the values assumed for the other form factors, to be taken from electron-nucleon data.

Salin also examines possible ways of determining the neutron form factors only from electroproduction and suggests a measurement of the transverse part of the cross-section, and observation of the recoil proton in  $\pi^0$  electroproduction.

The measurement of the longitudinal and transverse parts of the electroproduction cross-section would also be important for the evaluation of the neutron-proton mass difference in the dispersive formulation of this problem<sup>(47)</sup>. The relevant expressions have been recently produced by Cottingham<sup>(48)</sup>.

It would also be of interest to measure pion production from muons,  $\mu + p \rightarrow \mu + p + \pi$ , which could give information on the muon electromagnetic interactions. This process was studied some time ago by Von Gehlen<sup>(49)</sup>, who discussed in detail its phenomenological relation to electroproduction.

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